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## SURVEY ON HEAT TRANSFER AT HIGH SPEEDS

ERNST R. G. ECKERT

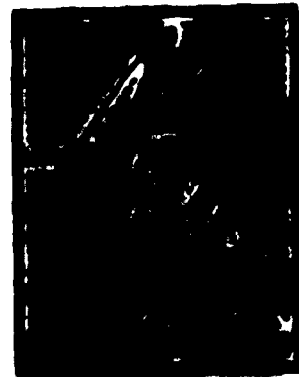
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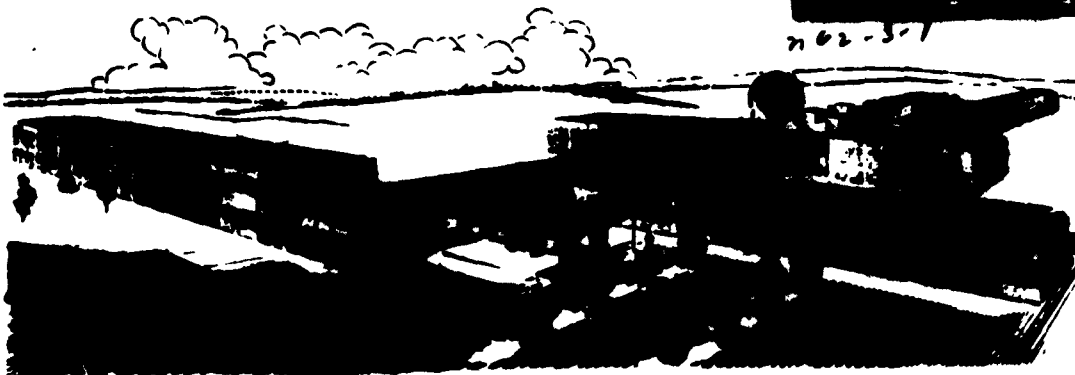
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AERONAUTICAL RESEARCH LABORATORY  
OFFICE OF AEROSPACE RESEARCH  
UNITED STATES AIR FORCE



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# **SURVEY ON HEAT TRANSFER AT HIGH SPEEDS**

**ERNST R. G. ECKERT**

**UNIVERSITY OF MINNESOTA  
MINNEAPOLIS, MINNESOTA**

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UNITED STATES AIR FORCE  
WRIGHT-PATTERSON AIR FORCE BASE, OHIO**

## FOREWORD

This technical report has been prepared by Dr. E. R. G. Eckert, Professor of Mechanical Engineering at the University of Minnesota under Contract AF 33(616)-2214 for the Aeronautical Research Laboratory, Office of Aerospace Research, Wright-Patterson Air Force Base, Ohio. The work reported herein was accomplished on Task 70138, "Investigation of Heat Transfer" of Project 7064, "Research on Aerodynamic Flow Fields". Mr. Erich Soehngen, ARL, was the project scientist.

This technical report supersedes WADC TR 54-70 dated April 1954.

Appreciation is extended to Miss Carolyn Byers for typing the manuscript.

## ABSTRACT

This report is a reissue of WADC TR 54-70 which presented a survey of the literature on heat transfer at high supersonic velocities published predominately in the years 1951 to 1954. The large number of papers made it necessary to restrict the survey essentially to surfaces along which the pressure is constant, an assumption which is not too restrictive since evidence could be presented that for slender shapes as generally used in high speed aircraft the influence of a pressure variation on heat transfer is small. Simple relations have been collected or developed which permit the calculation of friction factors, recovery factors, and heat transfer coefficients for laminar boundary layer flow along surfaces of constant temperature with an accuracy better than 3 per cent. A greater uncertainty in the prediction of the above parameters is caused by the poor knowledge of Prandtl number values for air at high temperatures. A decided advantage of the relations presented in this report lies in the fact that they appear to hold for any reasonable Prandtl number variation.

Relations for friction, recovery, and heat transfer have also been recommended for turbulent boundary layers. A method has been described by which the effect of an arbitrarily varying temperature along the surface can be established. The influence of the molecular structure of air appearing at flight at high altitude and of dissociation occurring at high flight velocities has been discussed. From this discussion specifically as well as from the survey generally the regimes become apparent in which future research is of special engineering importance.

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## SECTION I

### INTRODUCTION

The purpose of the present report is to present a survey and summary of presently available information on heat transfer at high supersonic speeds. It is hoped that the paper is prepared in such a way that it is useful for the designer who, in order to calculate temperatures in aircraft at high speeds, has to have information on the convective heat transfer between the aircraft skin and the atmosphere. It is well known that by the aerodynamic heating effect aircraft which fly with supersonic speeds, are heated up to such an extent that the skin temperature and the temperature inside the aircraft require the special attention of the designer.

Corresponding to the growing importance of cooling problems in aeronautics, the number of papers which deal with high velocity flow has steadily increased during recent years. This is evidenced by the fact that to-day, for example, each issue of the "Journal of the Aeronautical Sciences" contains at least one contribution to the heat transfer problem. In this situation a summary has to be restricted in its scope. In this report only convective heat transfer will be discussed and this only for surfaces along which the pressure is constant. The literature collected in the appendix is thought to be complete in this field. Such a restriction is permissible since aircraft flying through the atmosphere at high speeds will always have very slender shapes so that the pressure variation along the surface is not large and its effect on heat transfer is of minor importance. Some remarks on the influence of pressure variation are however included. Excluded also from the discussion will be surfaces which are cooled by a process known as transpiration cooling. In this process the skin is manufactured from a porous material and a cooling medium is blown through the skin into the atmosphere and builds up a cool film which protects the surface from the influence of the hot boundary layer. Information on this cooling process and its influence on heat transfer under high velocity conditions is still meager (Ref. 1, 2, 3, 4). The discussion is also restricted to heat transfer under steady state conditions. It has been shown that for rates of change in speed as they are usually encountered in aircraft the convective heat transfer may be regarded as quasi stationary (See Appendix IIL). This means the heat transfer at each instant may be calculated with formulae valid for steady state into which velocity and property values are introduced as they are encountered at this specific instant (Ref. 5, 6).

The essential features of the temperature distribution as it is encountered near a wall moving with high speed may be discussed with the help of Figure 1. In this figure the temperatures are plotted over the distance from the wall. In any high speed gas flow two different temperatures have to be distinguished: the static temperature  $T$  as it would be measured by an instrument which is at rest relative to the flow at each location, and the total Temperature  $T_t$  measured by an instrument which slows the flow adiabatically down to the velocity zero, measured relative to the wall. The full lines represent the temperature distribution for the case that no heat is conducted from the surface into the interior of the wall or transferred to the environment by radiation. The profiles of both temperatures are shown in the figure.

The static temperature increases throughout the boundary layer in the direction toward the wall because of the heat generated by internal friction. Under the conditions as specified no heat flow from the wall into the fluid occurs and therefore the temperature gradient at the wall is zero. However, the wall

temperature is higher than the static temperature in the stream outside the boundary layer because of the effect of "aerodynamic heating".

It is interesting to observe the behavior of the total temperature throughout the boundary layer. This temperature measures the total energy content of the fluid, internal and kinetic, and this energy content does not change in a steady flow from which energy is not extracted, either as heat or as mechanical energy. Both conditions are fulfilled by the boundary layer as a whole. Therefore, the average value of the total temperature has to be the same within the boundary layer as in the outside flow. When a defect of the total temperature in the layers near the wall is encountered for fluids with a Prandtl Number smaller than one, then it has to be compensated by excess of this temperature in the outer region of the boundary layer. For fluids with Prandtl number larger than one, the conditions are reversed, the total temperature being larger near the wall and smaller in the outer region of the boundary layer than in free stream.

The difference between the total and static temperatures at any point is determined by the local velocity  $V$ . For a fluid with constant specific heat  $c_p$  this difference is given by the expression shown in the diagram. For variable specific heat the difference between the corresponding enthalpies  $i_t - i$  is equal to the kinetic energy of the air  $V^2/2$ . For gases under the condition of no heat transfer, this wall temperature which will be called in this report the "recovery temperature" is almost equal to the total temperature in the stream outside the boundary layer. This leads at supersonic velocities to very high wall temperatures. Fortunately, conditions are usually much more favorable on the skin of aircraft flying at high speed because heat is radiated away from the surface of the aircraft or, for short flight duration, heat is being stored in the skin or other structural elements. The temperature profile found in the boundary layer under such circumstances is shown by dashed lines in Figure 1. It is interesting to note the peculiar shapes which the total temperature profile may assume. This profile is the one which can be comparatively easily measured by total temperature probes, whereas today no satisfactory direct measurement of local static temperatures is possible.

From the discussion of Figure 1 it is apparent that heat transfer in high speed flow is essentially influenced by two parameters which are usually not present in other heat transfer problems: a marked effect of frictional heating and a large temperature variation which occurs in the neighborhood of surfaces in a high velocity flow field, even when the surface itself is cooled down to the outside stream temperature.

It will be shown later, that for the case when the wall-temperature is cooled to the stream static temperature a maximum temperature within the boundary layer arises which for a Mach number 10 is five times the stream temperature or, for a Mach number 20 is equal to 18 times the stream temperature. These temperature variations are unusually large and cause the property values to change extensively throughout the boundary layer. The temperature within the boundary layer may become so large that it causes dissociation of the air.

Frictional heating as determined by the velocity of the air stream, large temperature variation, and high temperatures are the parameters which will have to be discussed extensively in this report with regard to their influence on heat transfer.

Another condition which is connected with aircraft application is the low pressure and correspondingly the low density of the air, encountered by the aircraft at high flight altitude. The air at high altitudes is so rarified that the lengths of the paths which the single air molecules travel between collisions become considerable and of the same order of magnitude as the thickness of the boundary layer. This changes heat transfer conditions from what is usually experienced in air flow under normal pressure. This factor will also be discussed in its influence on heat transfer in one of the following chapters.

## SECTION II

### CONVERSION OF FRICTION AND HEAT TRANSFER RELATIONS FOR WEDGES, CONES, AND CYLINDERS INTO THE RELATIONS FOR THE FLAT PLATE

Section III will deal extensively with the friction and heat transfer conditions as they exist in laminar or turbulent boundary layers on surfaces in two-dimensional flow for which the pressure does not change in flow direction. Such conditions are realized on a flat plate parallel to the stream direction as long as the boundary layers are very thin so that the displacement of the flow by this boundary layer can be neglected.

Constant pressure is in supersonic flow also encountered along the surface of wedges and cones when the shock waves by which the oncoming flow is turned into a direction parallel to the surface are attached to the vertex of the object. For this case simple conversion formulas exist by which friction and heat transfer data for cones and wedges can be obtained from information available for flat plates. These will be discussed in this chapter. Such conversion rules are important because parts of missiles or aircraft have conical shapes and also because many test results are available on cones and wedges and can in this way be compared with flat plate data.

Friction and heat transfer on the surface of a cylinder in a flow parallel to its axis are identical with the ones on a flat plate as long as the smallest radius of the curvature of the cylinder cross section is large compared with the boundary layer thickness. Some investigations as to the limiting conditions for this analogy will also be discussed.

For a wedge in a supersonic flow which is directed perpendicular to its edge and which has attached shock waves the flow conditions are constant in the field between the shocks and the surface of the wedge. Constant Mach number, constant total and static temperatures and constant density prevail in this field. Since the flow on such a wedge is two-dimensional and since the pressure along the surface of the wedge is constant, the relationship for skin friction and heat transfer, valid for flat plates, may be immediately applied to the surfaces of such a wedge. Conditions between the shock and the surface correspond to the flow conditions at the flat plate outside the boundary layer.

A cone may now be considered which is located in a supersonic flow directed parallel to the cone axis. The onstream conditions may be such that the shock wave is attached to the apex of the cone. The shock has in this case a conical shape with axis and apex coincident with the solid cone. Stream conditions in this case are not constant in the field between this shock and the cone surface, however, they are constant along any conical surface with the same axis and apex as long as the displacement of the flow by the boundary layer is negligible. For thin boundary layers, therefore, this condition will also hold along a conical envelope very near to the cone surface and just outside the boundary layer. When friction or heat transfer data from flat plates are to be converted to that on the surfaces of cones then the flow conditions, existing in this envelope which is practically coincident with the cone surface, correspond to the flow conditions for the plate. Again the pressure is constant along the surface in flow direction.

The flow, however, is not two-dimensional and therefore conversion formulas have to be used in order to obtain from flat plate data relationships valid for the surface of cones. Hantzsche and Wendt (Ref. 12) show by a transformation of the laminar boundary layer equations for flow along the cone surface to the flat plate case that the temperature recovery factor remains unchanged by this transformation and has the same value for cones as for flat plate. Local friction coefficients and heat transfer coefficients, valid for the flat plate, have to be multiplied by  $\sqrt{3}$  to obtain friction and heat transfer coefficients on the surface of the cone at the same distance from the leading edge and under identical stream conditions. This fact may also be stated in a different way, namely, that local friction and heat transfer coefficients are identical for cones and flat plates at locations for which the cone Reynolds number is times the value of the flat plate Reynolds number. At such corresponding locations within the boundary layer are completely identical and therefore not only the heat transfer coefficient and friction factors but also the stability conditions within the boundary layer and therefore the conditions for transition to turbulence are the same.

An analogous transformation on the turbulent boundary layer equations written in time mean values has been made by Gazley (Ref. 11), and in a very general form taking into account the variation of the property values by Van Driest (Ref. 9). The result of this transformation is that again conditions within the boundary layer are identical on such locations at the flat plate and on the cone surface for which the cone Reynolds number is equal to twice the flat plate Reynolds number. By the simple rules stated, friction and heat transfer can be calculated from formulas which are valid for flat plates.

For some purposes it is important to know also how much the local and the average friction and heat transfer values differ on the cone surface and the flat plate at locations with the same Reynolds number. For laminar flow it is already stated that the local heat transfer coefficient and friction factor on the cone are by the factor  $\sqrt{3}$  larger than on the flat plate. The average heat transfer coefficient and friction factor for the cone are larger by the factor  $2/\sqrt{3} \approx 1.15$ . It may be seen that the average friction and heat transfer values do not differ much for cones and flat plates when they are compared at the same Reynolds number. For the turbulent friction coefficient, using Blasius equation and Reynolds analogy, the following relationships for a turbulent boundary layer can be derived: The local friction and heat transfer coefficients are by the factor  $2^{1/4} \approx 1.15$  larger than for a flat plate. For the average values the factor is 1.022. Therefore, the difference between average friction and heat transfer coefficients on cones and flat plates considering locations with the same Reynolds number is only 2 per cent for turbulent flow. It may be added here that a transformation of the boundary layer equations for arbitrary bodies of revolution in rotationally symmetric flow to the two-dimensional case has been presented in Reference 14.

For a cylinder with circular cross section in a flow-subsonic or supersonic--which is parallel to the cylinder axis conditions within the boundary layer are principally different from the conditions on a flat plate since cylindrical areas within the boundary layer parallel to the surface increase in area proportional to the distance from the cylinder axis. The influence of this condition on heat transfer and friction in a turbulent boundary layer has been determined analytically by Jakob and Dow (Ref. 13), by H. U. Eckert (Ref. 10), and by Seban (Ref. 17). The latter investigator found that this condition does not influence the recovery factor so that flat plate recovery factors can be immediately applied to cylinders in axial flow.

With regard to the average skin friction the different predictions vary widely. An experimental investigation in reference 65, made on cylinder-cone combinations, found that at a Reynolds number around 10 an increase of the average friction factor of 5 percent was connected with a change of the length  $L$  to diameter  $d$  ratio from 8 to 23. The calculation by Jakob and Dow gives for a change of  $L/d$  from 8 to 23 an increase of 15 percent. H. U. Eckert's method results for the same conditions in 1.5 percent increase. The evidence presented does not give a conclusive quantitative information on how much friction and heat transfer on a cylinder are different from the ones on a flat plate. It can be concluded however, that up to a ratio of boundary layer displacement thickness to cylinder radius of 1:100 to 1:50, the difference between cylinder and flat plate data should be negligible.

For cylinders in a flow normal to their axis the condition for heat transfer are rather involved. A boundary layer builds up around the upstream part of the surface. The large pressure variation along the surface makes the development of this boundary layer quite different from the conditions considered before in this chapter. The effect of a pressure variation on the boundary layer will be considered in somewhat more detail in a later chapter. Heat transfer to the downstream part of the cylinder where the flow separates from the surface is still poorly understood. Extremely low recovery temperatures (partially below static upstream - temperature) have been observed in this region at subsonic upstream speeds. (Ref. 178, 179)

### SECTION III

#### HEAT TRANSFER TO A FLAT PLATE WITH CONSTANT SURFACE TEMPERATURE

This section deals with what is usually called the flat plate case. More exactly, it deals with heat transfer in two-dimensional flow, the pressure of which is constant along the surface. For thin boundary layers this type of flow is found on a flat plate. There are, however, special conditions like very low pressure, very high velocity where boundary layers become quite thick. The flow outside the boundary layer is then displaced and curved along a flat plate, according to the increase of the displacement thickness of the boundary layer in downstream direction. As a consequence of this, the pressure decreases in flow direction. Constant pressure in such a case would be rather obtained by a slightly concave surface. In supersonic flow, constant pressure along a surface is also found on wedges and cones with attached shocks. The simple rules with which flat plate formulas for heat transfer and friction can be transferred to the conditions on wedges and cones have been discussed in the previous chapter.

A constant temperature along the surface is also specified for this chapter. Generally, on missiles and aircraft the temperature will vary along the surface and it is known today that such a temperature variation has a considerable effect on heat transfer. The plate with constant temperature, however, serves as a standard condition to which comparatively simple formulas apply. A variation of the surface temperature has to be dealt with specifically by calculations in each individual case. This procedure will be discussed in a later chapter.

Conditions on the wall surface of the plate will be denoted by a subscript  $w$ , conditions in the stream outside the boundary layer by a subscript  $s$ . Here it is assumed that flow conditions in the field outside the boundary layer (velocity  $V_s$ , temperature  $T_s$ , density  $\rho_s$ ) are constant. The symbol  $T$  denotes absolute temperatures (in  $^{\circ}\text{R}$ ). In the flow  $T$  means always static temperature. The total temperature (as measured by a total temperature probe at rest relative to the plate) is indicated by a subscript  $t$ . The following dimensionless flow parameters are used:

$$Re = \frac{\rho_s V_s X}{\mu_s} \quad (B - 1)$$

$$Ma = \frac{V_s}{a_s} \quad (B - 2)$$

The Reynolds number  $Re$  is based on the distance  $X$  from the leading edge of the plate. The property values  $\rho$  and  $\mu$  in the Reynolds number will be introduced at different locations (wall or stream), whereas the Mach number  $Ma$  is always based on the sound velocity  $a_s$  at stream temperature.



The local shearing stress  $\tau_w$  which the flow exerts on the plate surface is expressed by a dimensionless friction factor defined in the following way

$$\tau_w = c_f \cdot \rho \cdot \frac{V_\infty^2}{2} \quad (B - 3)$$

The local convective heat flow per unit time and area at the plate surface is described by a heat transfer coefficient  $h$  which is defined in the following way

$$q_w = h (T_r - T_w) \quad (B - 4)$$

$T_w$  is the actual wall temperature and  $T_r$  is the temperature which the specific location on the surface assumes when the convective heat transfer is zero. This temperature will be called "recovery temperature" since it is conventionally described by the temperature recovery factor Eq. (Ba-4). The heat transfer coefficient is in turn expressed non-dimensionally by the Nusselt number

$$Nu = \frac{h \cdot x}{k} \quad (B - 5)$$

or by the Stanton number

$$St = \frac{h}{\rho \cdot c_p \cdot V_\infty} \quad (B - 6)$$

#### LAMINAR BOUNDARY LAYER FLOW

The differential equations which describe the flow and energy conditions for a laminar boundary layer are well established under normal pressure conditions in the continuum flow regime. Numerous solutions of these differential equations have been worked out and are today available to give information in a wide range of conditions, extending for instance to very high Mach number and large temperature differences. These results generally check satisfactorily with experimental investigations, when the experiments were made at the boundary conditions which correspond to the boundary layer solutions. Therefore, in the laminar range, the most reliable information can be obtained from calculations. The main difficulty in solving the differential equations is connected with the fact that the property values (viscosity, heat conductivity, density, and specific heat) vary for all gases and fluids with temperature and pressure. It is quite difficult to take into proper account the actual variation of the property values in the solution of the differential equations. The flat plate case is in this respect simpler than flow and heat transfer for general conditions. At the flat plate, as mentioned above, the pressure is constant along the surface. In the thin boundary layer the pressure also does not change in a direction perpendicular to the surface so that it is then constant over the whole field in which the flow and energy transfer occur. In consequence of this fact only the variation of density with temperature has to be accounted for in solutions of the boundary layer equations for the flat plate. In this connection it might be pointed out that the terminology which has become customary, namely to call a boundary layer on a flat plate which develops under high flow velocities a "compressible boundary layer" is not too well chosen. The established meaning of the term in physics is a change of volume in consequence of a change in pressure and such a change does not occur in a boundary layer on a flat plate since the pressure in the whole field of interest is constant. This expresses

itself in the fact that the character of the boundary layer equations is exactly the same for subsonic as in the supersonic range.

### Constant Property Solutions

The first solutions of the laminar boundary layer equations were obtained with the assumption that the property values in these equations are constant (independent of pressure and temperature). Under this condition the local skin friction factor was found to be

$$c_f = 0.664 / \sqrt{Re} \quad (Ba - 1)$$

The local Nusselts number was determined as

$$Nu = 0.332 \sqrt{Re} \sqrt{Pr} \quad (Ba - 2)$$

when the heat transfer coefficient  $h$  on which the Nusselt number is based is defined by the following equation

$$q_w = h (T_r - T_w) \quad (Ba - 3)$$

in which  $T_r$  is the temperature which the plate assumes when it is only in convective heat exchange with the high speed flow but otherwise not cooled or heated (by radiation, or by heat conduction into the solid wall interior). This wall temperature is in high speed flow larger than the static temperature in the stream outside the boundary layer because of the aerodynamic heating effect within the boundary layer and its value has to be known if the heat flow on a surface under high speed conditions is to be calculated. The calculations which were made under the assumptions of constant property values showed that the following temperature ratio:

$$r = \frac{T_w - T_s}{T_r - T_s} = \frac{T_r - T_s}{V_\infty^2 / 2 \cdot c_p} \quad (Ba - 4)$$

which is called temperature recovery factor is a function of only the Prandtl number of the fluid. This function is shown over a wide range of Prandtl numbers in Figure 2. (Ref. 37). It can be approximated by:

$$r = \sqrt{Pr} \quad (Ba - 5)$$

in the range of Prandtl numbers between 0.5 and 5. The wall temperature  $T_w$  which is described by the temperature recovery factor will in this report be called "recovery temperature".

The heat transfer coefficient is often expressed in a dimensionless form by a parameter called Stanton number:

$$St = h / \rho \cdot c_p \cdot V_\infty \quad (Ba - 6)$$

It can easily be seen that the following connection exists between the Nusselt number and the Stanton Number:

$$St = Nu / Re \cdot Pr \quad (Ba - 7)$$

A comparison of Equation Ba - 1 for the skin friction factor and Equation Ba - 2 for the local Nusselt number results in the following:

$$St = 0.5 c_f Pr^{-1/3} \quad (Ba - 8)$$

This relation between skin friction and heat transfer is useful because it still holds under certain conditions when the property values vary with temperature. This will be discussed in the next paragraph.

Reynold's analogy between skin friction and heat transfer can be expressed by the following equation:

$$St = \frac{c_f}{2} \quad (Ba - 9)$$

The heat transfer calculated from Reynold's analogy therefore differs from the correct relationship for laminar flow by the factor  $Pr^{-2/3}$ . For gases the Prandtl number is not too different from 1 and therefore Reynolds analogy gives a reasonable first approximation to the real heat transfer value. The relationships which have been determined for constant property values can be expected to hold satisfactorily for real fluids or gases as long as the temperature variation throughout the boundary layer is comparatively small. It has been found that this is the case as long as the ratio of wall to static free stream temperature is not too large (say between 0.5 and 1.5). Property data for determining the Reynolds and Prandtl numbers may be formed with temperatures  $T_w$  or  $T_\infty$ . The advantage of the constant property solutions is that they depend on a minimum of parameters and that they apply generally to any fluid.

#### Solutions For $\mu \sim k \sim T^\omega$

In the supersonic range and for large temperature differences the fact that the property values actually depend on temperature has to be accounted for in a solution of the laminar boundary layer equations. Figure 3 shows that the viscosity and heat conductivity vary considerably faster with temperature than the specific heat and the Prandtl number. Consequently the next step was to obtain boundary layer solutions for variable viscosity and conductivity but constant Prandtl number and specific heat. The equation

$$\frac{\mu}{k} = \frac{\mu}{c_p} \quad (Ba - 10)$$

shows that for constant Prandtl number and specific heat, the heat conductivity varies proportionally to the **viscosity**. Therefore the law of temperature dependence has to be prescribed for one property only. The following law was assumed to express the temperature dependency of the viscosity

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^n \quad (\text{Ba} - 11)$$

$\mu_0$  and  $T_0$  are the viscosity and temperature at some reference condition. The use of this law brings two advantages, first it simplifies the calculation procedure and secondly it makes the result dependent on a characteristic temperature ratio only, for instance, on the ratio of wall temperature, to free stream static or total temperature. This temperature ratio as well as the exponent  $n$  have to be added to the parameters mentioned above for the constant property value solutions.

Equation Ba - 11 expresses the actual temperature variation of the viscosity for gases and specifically for air reasonably well in a temperature range which is not too large. However, it is found that the numerical value of the exponent  $n$  depends on the temperature level. For very low temperatures  $n$  is approximately 1 and it decreases with increasing temperature asymptotically towards the value 0.5. Consequently, in different calculations different numerical values for this parameter were used. The following general results were obtained: (For a summary of these calculations see Reference 45). It was found that the expression Ba - 5 for the recovery factor still holds under the assumptions of this paragraph. In the same way the relationship Ba - 8 between skin friction and heat transfer is still valid. On the other hand it was noted that the skin friction and the heat transfer vary now with Mach number, with the characteristic temperature ratio, with the value of the parameter  $n$ , and the temperature level. Only the variation of the friction coefficient has to be discussed because of the validity of equation Ba - 8. Specifically the variation of the skin friction coefficient with the mentioned parameters depends on the way in which they are defined. From equations B - 1 and B - 3 which define the skin friction coefficient and the Reynolds number it can be seen that for variable property values a specific temperature must be chosen at which these values are introduced into the right hand side of the equations.

Usually the freestream static temperature is chosen as such a temperature. In this case the skin friction coefficient is found to decrease considerably with increasing Mach number, with increasing temperature ratio  $T_w/T_\infty$ , and with decreasing value of the parameter  $n$ . Figure 4 taken from reference 143 shows the dependence of the friction factor on Mach number. The dashed curves and the calculations by Van Driest, and Young and Janssen will be discussed later on. It is interesting to note that a specific value  $n = 1$  makes the dependence of friction factor on  $Ma$  and  $T_w/T_\infty$  vanish. The relations Ba - 1 to Ba - 9 hold, therefore, not only for constant properties but also when viscosity and heat conductivity vary proportionally to the absolute temperature. This fact adds considerably to the importance of the constant property solutions.

If the property values in Equations B - 1 and B - 3 for the skin friction factor for  $\text{Ma}$  different from one are introduced at the wall temperature, then the trend of the variation of the skin friction factor with Mach number is opposite to the one mentioned above.

It may therefore be expected that property values introduced at a proper reference temperature situated somewhere between the temperature extremes encountered within the boundary layer will cause the variation of the friction factor with  $\text{Ma}$  and  $T_w/T_s$  to vanish. Actually, Rubesin and Johnson (Ref. 49) have shown that it is possible to find such a reference temperature  $T^*$  and that it can be expressed with good accuracy by the following equation taken from Rubesin's and Johnson's paper:

$$\frac{T^*}{T_s} = 1 + 0.032 (\text{Ma})^2 + 0.58 \left( \frac{T_w}{T_s} - 1 \right) \quad (\text{Ba} - 12)$$

This expression for a reference temperature can also be transformed into the following:

$$T^* - T_s = 0.58 (T_w - T_s) + 0.19 (T_r - T_s) \quad (\text{Ba} - 13)$$

In this form the relation for the reference temperature appears especially instructive, as can be seen for some special cases sketched in Figure 5:

For low flow velocities the recovery wall temperature  $T_r$  is practically equal to the stream temperature  $T_s$ . The temperature profile in this case has the shape indicated in the upper diagram of the figure and Equation Ba - 13 shows that in this case the reference temperature  $T^*$  is located approximately halfway between wall temperature  $T_w$  and stream temperature  $T_s$ . This agrees with the results of other investigations on heat transfer at low velocities.

For high velocities but a small rate of heat transfer the wall temperature  $T_w$  is approximately equal to the recovery temperature  $T_r$ . The temperature profile is for this case indicated in the center portion of the figure and Equation Ba-13 shows that the difference between reference temperature  $T^*$  and stream temperature  $T_s$  is now 77% of the difference between wall temperature  $T_w$  and stream temperature  $T_s$ . Since the fluid layers near the wall will influence the heat transfer to the wall most strongly and since these layers have now, in an average, a higher temperature than in the first case considered, the value for the reference temperature is very credible.

As a third example let us assume that the wall is cooled to the stream temperature as indicated in the lower diagram of the figure. It will be seen later on that in this case the difference between the maximum temperature  $T_m$  within the boundary layer and the stream temperature is approximately 25% of the difference between recovery temperature and stream temperature. For the difference between reference temperature and stream temperature the corresponding value is 19%, which is again possible. It should be pointed out that Figure 4 presents friction factors for the case where the wall is at recovery temperature. For a cooled wall the friction factors are higher and for a heated wall lower than the values in the figure. Calculations of the friction factor with the reference temperature as given by Equations Ba - 12 and Ba - 13 properly accounts for these variations as will be shown in the following chapter.

## Solutions for Air with Actual Temperature Variation of All Properties

For very high velocities as are expected in future aeronautical applications the temperature variation throughout the boundary layer becomes so large, even in the case where the wall is cooled down to practically the free-stream static temperature, that it is doubtful how valid the results of calculations are which are based on the assumptions mentioned in the preceding paragraph. Accordingly, a number of recently published papers consider the specific heat and the Prandtl number as well as the heat conductivity and the viscosity to vary with temperature. All of the calculations have been made for air and more accurate relationships than a power law for the variation of the viscosity and the heat conductivity with temperature have been assumed.

Sutherland's relationship:

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{1/2} \frac{1 + C/T_0}{1 + C/T}$$

(Ba - 14)

with the constant  $C = 392^\circ \text{R.}$  for air, or tabulated values are used to express the viscosity. Tabulated values are used for the specific heat and Prandtl number. The conductivity is again fixed by equation Ba - 10. The main difference in the various sets of calculations is caused by the fact that different laws for the Prandtl number dependence on temperature have been used. Klunker and MacLean (Ref. 41) as well as Young and Janssen (Ref. 59) use Prandtl number data as contained in the Gas Tables and in a paper by Tribus and Boelter (Ref. 177). Van Driest (Ref. 36) on the other hand bases his calculations on Prandtl number values which have been recommended by the National Bureau of Standard (Ref. 156). Figure 3 indicates the Prandtl numbers as used in both sets of data. It may be recognized how large the differences in both sets of Prandtl number data are. All of the calculations had to extrapolate Prandtl number values to larger temperatures. This is indicated by the dashed part of the lines. The large differences in both sets of data reflect the lack of accurate knowledge of heat conductivity and Prandtl number for air at higher temperatures and is caused by the fact that it is extremely difficult to measure heat conductivity values for gases at larger temperatures.

The calculations mentioned above reflect the real conditions as encountered by an aircraft in the atmosphere better than the ones discussed in the previous paragraph since they are based on more accurate property value data; however, they have one disadvantage, namely, that flow and heat transfer now appear to depend on one additional parameter namely, on the temperature level in the air stream. The governing parameters are now Reynold's number, Mach number, Prandtl number, a characteristic temperature ratio and a characteristic temperature, for instance wall to stream temperature ratio, and stream temperature. The number of parameters is considerable and correspondingly the calculations become rather extended.

All the calculations show that the friction factor varies proportional to the square root of the Reynold's number in the same way as in Equation Ba - 1. Instead of the constant 0.664 in this equation, however, a function of Mach number, temperature ratio and temperature level appears. The most extensive calculations of this nature have been done by Young and Janssen (Ref. 59). The authors calculated friction factors for a wide range of Mach numbers (up to 30) for a large range of wall to stream temperature ratios (1.5 to 22) and for three values of the static stream temperature (100, 400, 800° R.). They also determined a reference temperature such that introduction of the property values at that temperature makes the value  $C_f$  equal to the incompressible value 0.664. Young and Janssen recommended Equation Ba - 12 for  $Ma$  up to 5 and give the following relationships which approximate the calculated reference temperatures well in a Mach number range from 5 to 10:

$$\text{for: } T_w = T_r : T^*/T_s = 0.0556 [13(T_r/T_s) + 6] \quad (\text{Ba} - 15)$$

$$\text{for: } T_w = T_r : T^*/T_s = 0.70 + 0.023 (Ma)^2 + 0.58 (T_w/T_s)$$

Since it is inconvenient to use different equations for different Mach number ranges and for the wall at recovery temperature and at a different value, it was investigated whether the reference temperatures can be expressed in the whole Mach number range by one relationship. The calculations presented in Appendix I resulted in the following equations for the reference temperature

$$T^* = T_s + 0.5 (T_w - T_s) + 0.22 (T_r - T_s) \quad (\text{Ba} - 16)$$

The equation may also be written as follows:

$$T^* = 0.5 (T_s + T_w) + 0.22 r \frac{C_p}{C_p^*} (Ma)^2 T_s \quad (\text{Ba} - 16a)$$

with  $r$  indicating the recovery factor and  $\frac{C_p}{C_p^*}$  the ratio of specific heats.

Table 1a contains in the fourth column the friction parameters  $C_f$  taken from Ref. 59. In the reference the property values were based on the stream temperature. The fifth column contains the same friction parameter as calculated with the reference temperature given in the last equation in the following way:

Equation Ba - 1 was assumed to hold when the properties are based on the reference temperature

$$(C_f Ma)^* = 0.664 = \frac{C_p}{C_p^*} \sqrt{\frac{2}{\gamma}} \quad (\text{Ba} - 17)$$

1	2	3	4	5	6	7	8	9	10	11	12
Run Nr.	T <sub>s</sub> (R)	L <sub>sa</sub>	(C <sub>p</sub> /He) <sub>s</sub>	(C <sub>p</sub> /He) <sub>s</sub>	%	T <sub>h</sub>	T <sub>h</sub>	%	r <sub>t</sub>	(Pr*) <sup>1/2</sup>	%
3c	99	4.23	0.690	0.685	-0.7	400	405	1.7	0.845	0.864	-2.2
4j	101	6.42	0.651	0.650	-0.2	800	793	-1.0	0.839	0.834	0.6
2d	99	8.17	0.620	0.619	-0.2	1182	1155	-2.5	0.828	0.820	1.
5a	100	9.57	0.593	0.592	-0.2	1559	1506	-3.0	0.807	0.810	-0.4
1d	104	11.67	0.555	0.555	-0.0	2246	2134	-5.3	0.807	0.803	0.5
6a	102	15.45	0.501	0.505	0.8	3565	3322	-7.0	0.798	0.797	0.1
7a	90	20.45	0.462	0.470	1.7	5135	4870	-5.1	0.781	0.793	-1.5
3e	400	0.22	0.674	0.664	-1.5	400	403				
4i	402	2.43	0.629	0.630	0.2	800	791	-2.2	0.840	0.827	1.6
2e	400	3.47	0.601	0.600	-0.2	1182	1159	-3.0	0.828	0.815	1.6
5b	401	4.28	0.575	0.576	0.2	1556	1516	-3.5	0.813	0.809	0.6
1e	405	5.52	0.541	0.542	0.2	2246	2154	-5.1	0.806	0.803	0.4
6d	401	7.49	0.499	0.495	-0.8	3565	3366	-6.3	0.796	0.797	-0.1
7b	388	9.57	0.456	0.456	-0.4	5135	4898	-4.8	0.780	0.792	1.5
8e	398	12.44	0.419	0.414	-0.7	8160	7868	-5.7		0.786	
9e	396	15.95	0.383	0.375	-2.1	12645					
1e	794	0.26	0.671	0.664	-1.0	800					
2f	800	1.74	0.638	0.635	-0.5	1182	1190	2.1	0.813	0.812	0.1
5e	808	2.45	0.612	0.610	-0.3	1550	1558	1.1	0.816	0.806	1.2
1f	797	3.52	0.576	0.574	-0.3	2246	2235	-1.0	0.788	0.803	-1.9
6f	810	4.95	0.533	0.527	-1.1	3565	3510	-1.6	0.806	0.797	1.1
7f	792	6.45	0.490	0.490	0.0	5135	5162	0.8	0.794	0.792	0.0
8e	805	8.54	0.448	0.443	-1.1	8160	8135	-0.3		0.786	
9f	765	11.33	0.405	0.399	-1.5	12645					

Some runs were not fully evaluated since the property values were not known for them.

Table Ia. Wall at recovery temperature.



1	2	3	4	5	6	7	8	9	10	11	12
Run Nr.	$T_{\text{OR}}$	$T_{\text{W}}$	Ma	$(C_f \sqrt{Re})_s$	$(C_f \sqrt{Re})_s$	error %	$\left(\frac{C_f}{2St}\right)^*$	$(Pr^*)^{1/4}$	error %	$\left(\frac{C_f}{2St_1}\right)$	error %
14d	97	400	8.63	0.667	0.661	-0.9	0.911	0.795	-14	0.934	-17
15d	101	400	11.84	0.629	0.634	-0.8	0.862	0.774	-11	0.924	-19
20f	94	400	19.19	0.584	0.583	-0.2	0.804	0.754	-6	0.878	-16
10f	101	1182	8.93	0.613	0.614	-0.2	0.873	0.766	-14	0.916	-19
11f	102	1182	12.37	0.594	0.589	-0.8	0.814	0.755	-7	0.876	-16
16f	105	1182	18.74	0.553	0.552	-0.2	0.776	0.746	-4	0.835	-12
13f	96	2246	13.39	0.553	0.547	-1.1	0.853	0.742	-14	0.898	-20
18f	80	2246	22.40	0.524	0.526	0.	0.799	0.742	-8	0.842	-14
10e	402	1182	4.55	0.591	0.590	-0.2	0.722	0.758	5	0.758	0
11e	405	1182	6.31	0.572	0.572	0.0	0.704	0.753	8	0.751	1
16e*	401	1182	9.70	0.538	0.534	-0.7	0.691	0.750	9	0.734	2
17g	404	1182	14.83	0.481	0.478	-0.6	0.686	0.733	7		
22e	410	1182	18.44	0.450	0.446	-0.9	0.692	0.731	6		
13e	398	2246	6.68	0.531	0.532	0.2	0.709	0.750	6	0.744	1
18e	394	2246	10.20	0.503	0.504	0.2	0.688	0.740	7	0.720	2.6
19e	394	2246	15.50	0.461	0.458	-0.6	0.692				
23e	416	2246	18.70	0.432	0.437	-1.1	0.695	0.730	6		
10d	798	1182	3.29	0.621	0.617	-0.6	0.726	0.758	4	0.755	0
11d	805	1182	4.55	0.602	0.604	0.3	0.718	0.750	4	0.756	-1
16d	792	1182	6.99	0.568	0.558	-1.8	0.708	0.750	6	0.745	1
17h	812	1182	10.57	0.513	0.510	-0.6	0.702	0.735	5		
22d	784	1182	13.43	0.476	0.477	0.2	0.677	0.731	8		
13d	801	2246	4.77	0.574	0.559	-2.6	0.725	0.750	3	0.755	-1
18d	804	2246	7.22	0.536	0.524	-2.2	0.672				
19g	800	2246	10.98	0.488	0.488	0.0	0.694	0.732	5		
23e	792	2246	13.63	0.462	0.464	0.4	0.685	0.730	6		

Some runs have not been fully evaluated, since the property values were not known for them.

Table Ib. Cooled wall.

The friction parameter based on stream conditions is then

$$(c_f \sqrt{Re})_s = \frac{c_{f0}}{9s} \sqrt{\frac{2}{\mu_s}} \sqrt{\frac{\rho_s V_s X}{\mu_s}} = 0.664 \sqrt{\frac{\rho_s V_s X}{9s \mu_s}}$$

(Ba - 18)

With the known values of the stream temperature and the reference temperature Equation Ba-18 can be used to determine the friction parameter as based on stream conditions. This parameter is compared in Tables 1a and b with the result of the calculations by Young and Janssen, and the error in per cent is entered into the sixth or seventh column. It is seen that the maximum difference between the friction factors calculated by both procedures exceeds only in three cases slightly the value 2%. Generally the error is probably within the accuracy of the boundary layer solutions. Figure 4 contains as dashed lines also friction parameters  $(C_f \sqrt{Re})_s$  which have been calculated with incompressible relationship Ba - 1 and the reference temperature as given by Equation Ba - 16 with the same procedure as used in connection with Table 1. It can be recognized that the agreement between the exact calculation and the simple calculation procedure using the reference temperature is excellent regardless of the specific law for the viscosity variation.

The papers which consider the temperature variation of all property values report on recovery factors which can not be expressed by the simple Equation Ba - 5. References 41 and 59 use the following definition for the temperature recovery factor

$$r_s = \frac{T_{r0} - T_s}{V_s^2 / 2 c_{ps}}$$

(Ba - 19)

Figure 6 shows these recovery factors and indicates a considerable variation with Mach number and stream temperature. This large variation of the recovery factor is surprising and it may be supposed to be caused by the fact that according to the definition Equation Ba - 19 the difference between recovery temperature  $T_r$  and static temperature  $T_s$  is compared with the difference between total temperature and stream temperature which a fluid would have if it had a constant specific heat equal to its real value at stream condition. Actually, the specific heat varies throughout the boundary layer and an average value should be used to determine the difference between total and static temperature. It may be calculated how well this average value for the specific heat is approximated by the specific heat which corresponds to the reference temperature given in Equation Ba - 16. Therefore the recovery values in Figure 3 were recalculated (Ref. 23) using the following definition for the recovery factor

$$r^* = \frac{T_r - T_s}{V_s^2 / 2 c_{p^*}}$$

(Ba - 20)

Table II.

$$T_s = 400^\circ R.$$

$T_w$	$M_a$	$(c_f \sqrt{K_s})_0$	$(c_f \sqrt{K_s})_1$	$(\frac{H}{\sqrt{K_s}})_0$	$(\frac{H}{\sqrt{K_s}})_1$
1182	4.55	0.591	0.598	0.311	-0.881
	6.31	0.572	0.738	0.319	-0.525
	9.70	0.538	0.684	0.325	-0.417
	14.83	0.481	0.752	0.313	-0.321
2246	6.68	0.531	1.460	0.312	-1.365
	10.20	0.503	1.085	0.319	-0.640
	15.50	0.461	0.967	0.305	-0.490

These recovery factors are also plotted in Figure 6 and it can be seen that most of the variation of the recovery factor with Mach number and stream temperature has disappeared. The remaining variation is well expressed by Equation Ba - 5 when the Prandtl number is introduced at the reference temperature described by Equation Ba - 16. This can be recognized from table 1a in which the recovery temperature as calculated by Young and Janssen is contained in column 7 and the recovery temperature determined with Equations Ba - 5 and Ba - 16 in column 8. The difference in per cent is entered in column 9. It exceeds only in a few cases 5 per cent.

Crocco already pointed out (Ref. 34) that the relationships obtained for a gas with constant specific heat remain materially unchanged for a gas with variable specific heat when the product  $c_p dT$  is replaced by the differential of the enthalpy  $i$ . In newer papers (Ref. 142 and 36) this fact is utilized.

Van Driest bases his recovery factors on enthalpy instead of temperature and uses the following definitions for the enthalpy recovery factor

$$r = \frac{i_r - i_s}{i_t - i_s} = \frac{i_r - i_s}{V_s^2 / 2} \quad (\text{Ba} - 21)$$

It can be observed that this definition is very similar to the one given in the above Equation Ba - 20. Figure 7 contains as solid lines the enthalpy recovery factors as calculated in Reference 36 for two different conditions in the stream outside the boundary layer. The upper graph of the figure presents the recovery factor for a stream with a static temperature of  $400^\circ\text{R}$ . The lower diagram gives the recovery factor for a total temperature in the stream equal  $100^\circ\text{F}$ . A static temperature of  $400^\circ\text{R}$  was chosen to approximate conditions as encountered by an aircraft in flight through the atmosphere whereas a total temperature of  $100^\circ\text{F}$  is supposed to represent conditions as found in wind tunnels. A comparison of the recovery factors in this figure taken from Ref. 59 with the enthalpy recovery factors in Table 1a discloses a marked difference especially at higher Mach numbers. This is due to the different temperature variation of Prandtl number on which the calculation by Young and Janssen on one side and by Van Driest on the other side is based. It was pointed out before, that the lower Prandtl number values were used in Ref. 59, whereas Van Driest adopted the set of higher values in Figure 3. Also inserted in the figure as dashed lines are recovery factors which have been calculated using Equation Ba - 5 into which the Prandtl number was introduced at a reference temperature as given by Equation Ba - 16. It may be observed that the agreement between the exact calculation and the calculation from Equation Ba - 5 is very satisfactory. The maximum deviation between both sets of calculations is less than 2 per cent. From the data published in Reference Ba - 28 the enthalpy recovery factors have been calculated and inserted in Table 1a as column 10. They can be compared with the values  $\sqrt{Pr}$  in column 11. The difference in per cent is entered into column 12. It may be observed that the difference is only in one case larger than 2 per cent. It is smaller than the error in column 6 for the temperature recovery factors. Use of the enthalpy recovery factor is therefore preferable. It is also remarkable that the value  $\sqrt{Pr}$  approximates the actual recovery factor so well regardless of the Prandtl number variation on which calculations have been based.

Heat transfer coefficients have been calculated by Young and Janssen for the same range of parameters as the friction factors. One result of this as of all other calculation procedures is that the Nusselt number as a dimensionless parameter for the heat transfer coefficient increases proportional to the square root of the Reynolds number. Accordingly, Young and Janssen (Ref. 59) presented their data as the parameter  $Nu / \sqrt{Re} \sqrt{Pr}$  in which all the property values have been introduced at the stream temperatures assuming that the Nusselt number is also proportional to  $\sqrt{Pr}$ .

Since it has been shown that the friction factor can be expressed very accurately by a simple formula, it appears advantageous to calculate the heat transfer coefficient with the help of the friction factor. Therefore the ratio  $c_p/2 St$  which according to Equation Ba - 8 has the value  $Pr^{1/3}$  for a fluid with constant property values, was determined from Young and Janssen's data. With property values based on stream temperature the following equation holds

$$\left(\frac{c_p}{2 St}\right)_s = \frac{(c_p \sqrt{Re})_s}{2 (Nu / \sqrt{Re} \sqrt{Pr})_s} (Pr)_s^{2/3} \quad (Ba - 22)$$

The parameters on the right side of the equation are tabulated in Reference 59. Now it will be checked how well Equation Ba - 8 approximates the values calculated by Young and Janssen when all properties in this equation are introduced at the reference temperature described by Equation Ba - 16. With Equations B - 3 and B - 6 the ratio friction factor to Stanton number can be written

$$\left(\frac{f}{2 St}\right)_s = \frac{c_p}{h V_s} \quad (Ba - 23)$$

The only property value in this equation is the specific heat  $c_p$ . Therefore the corresponding ratio based on properties at reference temperature is

$$\left(\frac{f}{2 St}\right)^* = \frac{c_p^*}{h V_s} = \left(\frac{f}{2 St}\right)_s \frac{c_p^*}{c_{p,s}} \quad (Ba - 24)$$

This parameter as calculated from Young and Janssen's data is contained in column 8 of Table 1b. The values  $Pr^{1/3}$  are contained in column 9 and the percentage difference in column 10. It can be observed that the agreement for the heat transfer parameter is not as good as for the friction factors and the temperature recovery factors. Only for the two stream temperatures 400°R. and 800°R. is it within 10 per cent. Calculations by Klunker and MacLean (Ref. 41 and 42) are less extensive than the one in Reference 59 but are generally in good agreement with the latter. Therefore a comparison with the incompressible relationship was not made for these calculations.

Van Driest\* has also calculated heat transfer coefficients with the Prandtl number variation as proposed by the National Bureau of Standards. He used in the presentation of his results a Stanton number which is based on enthalpies  $i$  instead of temperatures analogously as in the definition of the recovery factor. Instead of the customary definition of the Stanton number

$$St = \frac{q_w}{g c_p V_s (T_r - T_w)} \quad (Ba - 25)$$

he defines it in the following way

$$St_1 = \frac{q_w}{g V_s (i_r - i_w)} \quad (Ba - 25a)$$

The paper contains plots of the parameter  $c_p/2 St$ , which for constant Prandtl number has according to Equation Ba - 8 the value  $1/(Pr)^{1/3}$ . Figure 8 shows this parameter plotted over the Mach number for two different conditions in the free stream. The upper diagram holds for a free stream static temperature of  $400^\circ R$ , and the lower diagram holds for a total free stream temperature of  $100^\circ F. = 560^\circ R$ . Compared with these boundary layer solutions are again the results of an approximate procedure which sets the parameter  $c_p/2 St_1$  equal to  $Pr^*$  and introduces the Prandtl number at the reference temperature as given by Equation Ba - 16. It may be observed that the agreement between both sets of calculations is again excellent. The deviation is in the whole range investigated less than 1 per cent. In view of this good agreement the parameter  $c_p/2 St_1$  was also calculated from Young and Jonsson's data. It is contained in column 7 of Table 1b. Column 8 gives again the per cent difference between the actual values in column 7 and the approximation by Equation Ba - 8 as contained in column 5. It can be observed that the agreement is poor for a stream temperature  $T_s = 100^\circ R$  and  $800^\circ R$ . A stream temperature of  $100^\circ R$  may occur in wind tunnel tests but is never encountered in the atmosphere. For design calculations, therefore, the use of the Stanton number based on enthalpy is preferable. This Stanton number can be determined from Equation Ba - 8 with an error less than 2.6 per cent. When calculations of recovery factors or heat transfer coefficients are based on enthalpy, it is probably more convenient to determine the reference conditions at which the property values are introduced from a reference enthalpy instead of the reference temperature (Equation Ba - 16). For this reason the calculation procedure as described in Appendix I was repeated on the basis of enthalpies instead of temperatures. It was found that within the accuracy obtainable the equation

$$i^* = i_s + 0.5 (i_w - i_s) + 0.22 (i_r - i_s) \quad (Ba - 26)$$

\* see addendum on p. 45.

which has the same numerical constants as Equation Ba - 16, determines the reference conditions which, when used for the properties in the equation for the friction factor, gives best agreement with exact boundary layer solutions. The reference enthalpy may also be expressed in a different way as:

$$i^* = 0.5 (i_s + i_w) + 0.22 r_1 \frac{T-1}{2} (Ma)^2 i_s \quad (\text{Ba} - 26a)$$

A comparison of measured temperature recovery factors and heat transfer coefficients with calculated values has been made in a recent paper by G. Eber (Ref. 155). Eber shows that measured temperature recovery factors in a laminar boundary layer on cones vary around the value 0.85 with a maximum deviation of 1 per cent. Within this limit the values agree also with the calculated results which have been presented in this paper.

In the Figure 4 of his paper, Eber compared measure recovery factors with values which have been calculated by Klunker and McLean and points out that for larger Mach numbers a considerable difference exists. It has to be kept in mind, however, that the calculated data by Klunker and McLean have been obtained for free-stream conditions which were different from the ones for which the measurements have been made. The measured recovery factors have to be compared with the values in Figure 7 and exhibit then an agreement which again is within 1 per cent. The peculiar fact that temperature recovery factors measured on a flat plate were consistently higher than the ones determined from cones and assumed values around 0.88 has been explained by Korobkin (Ref. 43) as caused by heat conduction in the solid plate material. Heat transfer coefficients could only be measured with an accuracy of approximately 10 per cent and within this limit again agreement between values measured on cones and calculated heat transfer coefficients have been observed.

#### Recommended Calculation Procedure

On the basis of the investigation made in this chapter the following procedure is recommended for the calculation of the friction and heat transfer of flat plates in supersonic laminar flow:

The wall shearing stress is calculated from the following equation

$$\tau_w = c_f \frac{\rho^* V^2}{2} \quad (\text{Ba} - 27)$$

with the friction factor appearing in this equation given by the relationship

$$c_f = 0.664 / \sqrt{Re^*} = 0.664 / \sqrt{\frac{\rho^* V_\infty x}{\mu}} \quad (\text{Ba} - 28)$$

The property values are introduced into both of the above equations at a reference temperature which may be tentatively calculated from the following relationship

$$T^* = T_s + 0.5 (T_w - T_s) + 0.22 (T_r - T_s) \quad (\text{Ba} - 29)$$

This procedure was shown to give agreement with exact boundary layer solutions within  $\pm 2$  per cent.

Heat transfer coefficients are defined by the following equations:

$$q_w = h (T_r - T_w) \quad (\text{Ba} - 30)$$

for constant specific heat, or:

$$q_w = h_i (i_r - i_w) \quad (\text{Ba} - 31)$$

for variable specific heat.

The recovery temperature is determined by the relation

$$T_r = T_s + r \frac{V_s^2}{2 c_p} \quad (\text{Ba} - 32)$$

for constant specific heat and by

$$i_r = i_s + r_i \frac{V_s^2}{2} \quad (\text{Ba} - 33)$$

for variable specific heat.

The recovery factor appearing in both of the above equations is calculated from the equation

$$r = \sqrt{\text{Pr}^*} \quad (\text{Ba} - 34)$$

into which the Prandtl number has again to be introduced at the reference temperature as given by Equation Ba - 24. Alternatively the Prandtl number to be introduced into



Equation Ba - 34 can also be determined from the reference enthalpy

$$i^* = 0.5 (i_s + i_w) + 0.22 r_1 \frac{\gamma-1}{2} (Ma) i_s \quad (\text{Ba} - 35)$$

Recovery factors calculated in this way agree within  $\pm 2$  per cent with the results of exact boundary layer solutions.

The heat transfer coefficient appearing in Equation Ba - 30 and Ba - 31 calculated from the Stanton number value

$$St = \frac{h}{\rho c_p V_s} \quad (\text{Ba} - 36)$$

for constant specific heat or from

$$St_1 = \frac{h_1}{\rho V} \quad (\text{Ba} - 37)$$

for variable specific heat. The Stanton numbers are obtainable from the relationship

$$St = \frac{c_p}{2} (Pr^*)^{-2/3} \quad (\text{Ba} - 38)$$

into which the Prandtl numbers have again to be introduced at the reference temperature given by Equation Ba - 28 or the reference enthalpy Equation Ba - 35. Heat transfer coefficients calculated in this way agree within 2.6 per cent with the results of exact boundary layer solutions, for free stream temperatures above 300°R.

The main uncertainty in the calculation of recovery factors and heat transfer coefficients is caused by our insufficient present day knowledge of Prandtl numbers for air at higher temperatures. Prandtl number values as given in the Gas Tables and recommended by the National Bureau of Standards differ for instance at a temperature of 2000°R. by 12 per cent and accordingly the recovery factors calculated with those Prandtl numbers differ by 6 per cent and heat transfer coefficients by 8 per cent.

## TURBULENT BOUNDARY LAYER FLOW

In the preceding chapter it has been demonstrated that the calculation of friction and heat transfer in laminar boundary layer flow has been refined to an accuracy which is very difficult to parallel in experimental investigations. In turbulent boundary layers the situation is completely different. The present day understanding of turbulent exchange of momentum and energy especially under conditions of widely varying temperature is restricted to such an extent that exact solutions of the boundary layer equations can not be obtained. In any calculation procedure assumptions have to be introduced on the exchange mechanism and these assumptions have to be continuously checked with experiments. In obtaining friction and heat transfer data we have, therefore, to rely heavily on measurements. Calculations are necessary and useful in order to extend the range of parameters outside the experimentally investigated one. However, it will be necessary to select from the different calculations procedures published in the literature those which agree well with experimental data in the investigated range. Comparatively extensive experimental investigations are available for the friction factor of plates whose temperatures are equal to the recovery temperatures.

In Figure 9 (Ref. 65) the information on the influence of Mach number on this friction factor is collected. The measurements, the results of which are presented in the figure, have been made on flat plates or cylinders. Plotted is the ratio of the friction factor in compressible flow to the friction factor in incompressible (means constant property value) flow. The friction factor is again based on the property values taken at stream temperature. It may be observed that it decreases considerably with increasing Mach number. The influence of Reynolds number on the friction factor has been found to be the same for supersonic as for low velocity flow. Therefore Figure 9 applies to any value of the Reynolds number. Friction factors which have been measured on cones by Bradfield (Ref. 18) have values which are approximately 20 per cent higher than the results of the measurements collected in Figure 9. The reason for this discrepancy is not explained until now.

Figure 10 (Ref. 65) presents the predictions which different published theories make on the change of friction factors with Mach number. It may be observed that the various predictions differ to a very high degree. The best agreement to the experimental results presented in Figure 9 give calculations made by Wilson, Monaghan, Cope, Frankl-Voishel, and Tucker. But even with these results it is difficult to make predictions on the friction factor at higher Mach numbers because the different theories deviate considerably at Mach numbers above five.

Faced with this situation it is appropriate to look for a simple calculation procedure which agrees with the experimental results in the investigated range. Young and Janassen (Ref. 59) pointed out that the use of constant property relations with the reference temperature established for laminar boundary layers gives good agreement for turbulent boundary layers as well. For this procedure the relationship for skin friction in a constant property fluid flow must be established at first. Several equations are offered in the literature for this purpose.

The Blasius equation

$$\frac{c_f}{2} = 0.0296 \operatorname{Re}^{-0.2}$$

(Bb - 1)

gives good agreement with measured values up to  $Re = 10^7$ . At larger Reynolds numbers Equation Bb - 1 gives friction factors which are too low.

A good representation of the measured average friction factors in the whole range up to  $Re = 10^9$  is obtained by the Karman-Schoenherr equation

$$\frac{0.242}{\sqrt{c_f}} = \log_{10} (Re \sqrt{c_f}) \quad (\text{Bb} - 2)$$

The local friction factor may be calculated from the average value by the relation

$$c_f = \frac{0.557 \sqrt{c_f}}{0.557 + 2 \sqrt{c_f}} \quad (\text{Bb} - 3)$$

Simpler to apply is a formula by Schulz-Grunow for the local friction factor

$$c_f = \frac{0.370}{(\log_{10} Re)^{2.584}} \quad (\text{Bb} - 4)$$

and a relation by Prandtl-Schlichting for the average friction factor

$$\bar{c}_f = \frac{0.455}{(\log_{10} Re)^{2.58}} \quad (\text{Bb} - 5)$$

Both of these relations represent measurements up to  $Re = 10^9$  very well.

Since the measurements plotted in Figure 9 have been made at Reynolds numbers below  $10^7$ , the Blasius Equation (Bb - 1) will be used to establish the Mach number dependence of the friction factor with the use of the reference temperature. The friction factors in Figure 9 are based on property values at stream temperature

$$\tau_w = c_f \frac{\rho_s v_s^2}{2} \quad (\text{Bb} - 6)$$

and compared with the friction factor  $c_{f1}$  of an incompressible (constant property) fluid at stream temperature

$$\frac{c_f}{2} = \frac{0.0296}{(g_s v_s x / \mu_s)^{0.2}} \quad (\text{Bb} - 7)$$

Assuming that for a variable property (compressible) fluid the shearing stress is given by the constant property value relationship when the property values are introduced at the reference temperature  $T^*$  results in the equation

$$\tau_w = \frac{0.0296}{(g^* v_s x / \mu^*)^{0.2}} \frac{g^* v_s^2}{2} \quad (\text{Bb} - 8)$$

A comparison between the Equations Bb - 6, Bb - 7, and Bb - 8 gives the following formula by which the ratio of the friction factor in supersonic flow to the friction factor for a fluid with constant property value can be calculated.

$$c_f / c_{f1} = \left( \frac{\mu^*}{\mu_s} \right)^{0.2} \left( \frac{g^*}{g_s} \right)^{0.8} \quad (\text{Bb} - 9)$$

The results of such a calculation in which it was assumed that the viscosity varies proportional to the power of 0.76 of the absolute temperature is indicated by a dashed line in Figure 9. It may be observed that this dashed line averages the measured values very well. It also agrees at higher Mach numbers with the calculations by Wilson, Monaghan, and Tucker as presented in Figure 10. It can therefore be recommended for a plausible extrapolation of the test results into the higher Mach number range. A paper (by I. H. Abbot - Ref. 181) presented to the fourth meeting of AGARD wind tunnel and model testing panel contains the results of test on thin-walled tubes in a firing range. Two series of test points were obtained, one at Mach number 3.9 and for the wall cooled to stream static temperature, the other at Mach number 7.25 and  $T_w = 1.8 T_s$ . Both groups of test points are shown in Figure 9. The dashed line in this figure represents the ratio  $c_f/c_{f1}$  as calculated with Equations Bb - 9 and Bb - 16. The agreement is especially good for the smaller Mach number, for which according to Abbott the tests reproduced most satisfactorily.

Some friction coefficients on a cooled wall are also reported in Reference 180. They were measured at Mach numbers between 5 and 7.7. The values  $c_f/c_{f1}$  for these tests are all lower than even the curve for the wall at recovery temperature shown in Figure 9. It is believed that this is caused by the fact that the measurements were made on a wind tunnel wall along which the pressure was not constant but decreased in flow direction.

Reference 73 contains the results of a very extensive investigation which has been made at the NACA to study friction factors and heat transfer coefficients for air flowing turbulently through a round tube with large temperature differences and subsonic velocities. The essential result of these tests was that the normal constant property value relationship described the results of these tests satisfactorily when the property values were introduced at a reference temperature which was situated half way between the wall temperature and the air temperature. It may be observed that equation Ba - 16 contains the same relation for the reference temperature for the case that the air velocity is small (recovery temperature equal to wall temperature). Therefore, it may be justified to recommend use of constant property value relationships together with equation Ba - 16 as reference temperature for turbulent flow along a flat plate.

By plausible deductions Ackermann (Ref. 62) and Squire (Ref. 85) determined the following relationship for the recovery factor in a turbulent boundary layer

$$r = \sqrt[3]{Pr} \quad (Bb - 10)$$

Test results on flat plates as well as on cones agree very well with this prediction so that it is today in common usage to describe the temperature recovery in a turbulent boundary layer on a flat plate. The test results showed no conclusive variations of the recovery factor with Reynolds number or with Mach number. A slight decrease of the recovery factor with Mach number which has been predicted by an extension of Squire's calculation into the compressible flow range (Ref. 87) has up to now not been confirmed by experiments, probably, because this effect is too small in the range of Mach numbers investigated till now. It has to be remembered that all experimental information has been obtained at a total temperature of the free stream around 100°F.

The simplest way to calculate heat transfer in turbulent flow is again to derive it from the friction factor with an equation

$$St = \frac{C_f}{2} S \quad (Bb - 11)$$

analogous to relationship Ba - 8 in which the function S has to be determined experimentally.

For small temperature differences, and low velocities Colburn (Ref. 66) has found that this factor S can be expressed for turbulent flow as well as for laminar flow as  $1/Pr^{1/3}$ . Accurate test results for which this expression could be checked in high velocity flow with large temperature differences are not known to the author.

Rubesin (Ref. 82) obtained in a calculation in which he had again to make plausible assumptions on the turbulent exchange mechanisms the result that the factor  $S$  agrees for high velocity high temperature difference flow for a Reynolds number below  $10^6$  with the expression  $1/Pr^{1/3}$  and that it decreases slightly with Reynolds number so that for a Reynolds number of  $10^7$  it assumes for air the numerical value 1.20 and for a Reynolds number equal  $10^9$  the value 1.18. The influence of Mach number and of the wall to stream temperature ratio was in this calculation found to be extremely small.

Van Driest (Ref. 36) makes in his well known calculation of heat transfer in the turbulent high velocity boundary layer the assumption that the Prandtl number and the ratio of turbulent diffusivities for momentum and heat are both equal to one. For such a fluid Reynolds analogy is exactly fulfilled. This means that for  $Ma = 4$  the friction factor of a wall with the temperature equal to the recovery temperature as calculated by Van Driest is approximately 20 per cent larger than the one measured experimentally. It is therefore understandable that the heat transfer coefficient calculated by Van Driest agrees well with experiments and with the predictions of Rubesin.

All the discussion in this chapter assumed that the plate surface is smooth. It is known that roughness increases friction and heat transfer in turbulent flow when the roughness exceeds certain limits. Extensive systematic information on rough surfaces exists only for the friction coefficient in low velocity flow. A good survey over this field is for instance contained in Reference 27.

Summarizing our discussion on turbulent boundary layers the following recommendation is made for a calculation procedure: Constant property relationships should be used to calculate the friction factor, the recovery factor, and the heat transfer coefficient, namely Equations Eb - 1 to Eb - 5 for the friction factor, Equation Eb - 10 for the recovery factor, and Equation Eb - 11 for the heat transfer coefficient. The factor  $S$  in the latter equation varies for air between 1.25 and 1.18 in the Reynolds number range from  $10^5$  to  $10^9$ . Property values should be introduced into these equations at the reference temperature given by equation Ea - 16 and the recovery factor as well as the Stanton number describing the heat transfer should be based on enthalpies instead of temperatures when the temperature range is such that the specific heat varies considerably within the boundary layer. The recommendations have to be considered as tentative and have to be checked by more experiments at large Mach numbers and especially at large temperature differences and high temperatures.

## SECTION IV

### INFLUENCE OF PRESSURE AND TEMPERATURE VARIATION ALONG THE SURFACE ON HEAT TRANSFER

#### VARIABLE PRESSURE

Up to now the pressure was assumed to be constant along the surface of the body to which heat is transferred. Actually, the pressure usually varies along the body as well as along the airfoils of aircraft and it is important to know the influence of such a pressure variation on heat transfer.

Exact solutions of the boundary layer equations for laminar flow and for fluids with constant properties exist for certain types of velocity variation especially for a velocity outside the boundary layer which increases proportional to some power of the distance from the leading edge.

Approximate procedures which use the integrated momentum and energy equation have also been worked out for the calculation of heat transfer on surfaces along which the pressure varies in some arbitrary way. Information obtainable from calculations is much more restricted for high velocity flow in fluids with variable properties. Only in a very few cases solutions of the boundary layer equations have been obtained and even procedures which use the integrated boundary layer equations become rather tedious.

Generally it is known that a pressure variation influences heat transfer coefficients to a lower degree than the skin friction parameters. This is caused by the fact that the pressure gradient enters the energy equation only in an indirect way. In addition to this fact, the shapes, used for aircraft bodies and airfoils designed to fly at high velocities, have to be made very streamlined in order to reduce the drag. On such configurations which for supersonic flow have also sharp noses or leading edges the pressure variation is comparatively small and therefore the effect on heat transfer is of minor importance. This finding is substantiated by experiments contained in Reference 65 in which average heat transfer coefficients were measured on a cylindrical body with a conical nose. The pressure along the cylinder could be varied by an adjustment of the wind tunnel walls. In addition to a constant pressure along the surface, pressure variations were investigated corresponding to Mach number variations along the surface as shown in Figure 11. It is mentioned in the referenced report that such a variation corresponds approximately to the pressure variation as found around a 3 per cent thickness biconvex air-foil or to the pressure variation around a parabolic arc body of revolution with a length to diameter ratio equal to 15. It was found that for pressure variations as indicated in Figure 11 the average heat transfer coefficient varied only to a degree which was smaller than the accuracy with which heat transfer coefficients could be measured.

The same conclusion was reached in Reference 26 where local heat transfer values and recovery factors have been measured on cones with straight and parabolic generatrices. It can therefore be expected that for bodies with small angle noses or leading edges and for flow under small angle of attack the constant pressure relations give heat transfer data of sufficient accuracy for design purposes. With

respect to cooling a rounded nose would be advantageous for missiles since in this way the large heat transfer coefficients near leading edges are avoided. Heat transfer on such a rounded nose will decidedly be different from that on a flat plate. An extensive treatment of the influence of pressure variation on heat transfer to bodies of arbitrary shape would exceed the scope of this paper. For this reason no further discussion on the influence of pressure variations will be made and the reader is referred to the literature presented in the appendix.

One remark may be added, namely that for flow with pressure gradients the correlation of the actual friction and heat transfer parameters with the constant property relations by specification of a reference temperature is not possible. This can be seen from Reference 99 where calculations are made under the assumption that the product of density and viscosity is independent of temperature. Friction factor and Nusselt number are found to vary with the ratio wall to stream temperature. No such variation would be indicated by the constant property relation for any choice of the reference temperature.

#### VARIABLE WALL TEMPERATURE

That a temperature variation along the surface of a heated object is a parameter of importance for the heat transfer coefficient has been only recently generally recognized. Fage and Falkner (Ref. 115) presented for laminar flow an approximate treatment of this problem as early as in 1931 but it was only after 1950 that intensive work was started to investigate the dependence of heat transfer on this parameter. All calculations on heat transfer at variable wall temperature are based on the fact that for constant property values the energy equation which describes the heat flow through a boundary layer into a wall is linear and that therefore general solutions of this equation can be obtained by a superposition of individual special solutions. This linearity of the energy equation is in laminar flow essentially still preserved when the property values vary in such a way that the product of density times viscosity is constant. Under this condition and for a specific type of temperature variation along the surface:

$$T_w - T_r = C x^\beta \quad (Cb - 1)$$

Chapman and Rubesin (Ref. 113) obtained exact solutions of the laminar boundary layer equations describing the flow and heat transfer to a flat plate. Figure 12 contains heat transfer coefficients based on the local temperature difference  $T_w - T_r$ , as calculated in this reference. The parameter  $Nu/\sqrt{Re}$  is plotted over the exponent which determines the temperature variation along the surface. It may be observed that with increasing value of  $\beta$  a considerable increase in the local heat transfer coefficient as determined by the parameter  $Nu/\sqrt{Re}$  occurs. When the temperature difference between the wall and recovery temperature, which determines the heat transfer, increases linearly with distance from the leading edge the heat transfer coefficient will be by 65 per cent larger than the heat transfer coefficient under corresponding conditions but for a constant wall temperature. For a temperature difference which increases proportional to the square of the distance from the leading edge, the heat transfer coefficient is larger by a factor of 2 than the heat transfer coefficient for constant wall temperature.



Conditions similar to the ones just discussed occur near the nose or leading edge of a body or airfoil of an aircraft in high speed flight when the aircraft surface is cooled either by radiation or by heat storage in the solid material. In this case the wall surface temperature is determined by a balance between the heat transferred from the air to the surface by convection and the heat removed from the surface by radiation or by heat storage in the interior. It is known that the heat transfer coefficient on the leading edge of objects is very large (theoretically on the flat plate it is equal to infinity) and that it decreases with increasing distance from the leading edge. Correspondingly, the temperature of the object near the leading edge is practically equal to the recovery temperature and it drops with increasing distance because the heat removal through radiation becomes more and more effective as compared with the convective heat transfer. Temperature distributions in wings of diamond shaped as calculated by Kaye (Ref. 116) show this characteristic behavior. It has to be expected that the heat transfer coefficients in a close region downstream of the leading edge of such objects will be considerably larger than the heat transfer coefficients calculated under the assumption of a constant wall temperature. Correspondingly the surface temperatures will be higher.

It is possible to determine heat transfer for laminar flow conditions and for an arbitrary variation of the wall temperature by superposition of individual solutions which have been obtained for a temperature variation according to Equation Cb - 1 by expressing the temperature variation along the surface as a series:

$$T_w - T_r = \sum C_i x^{\beta_i} \quad (\text{Cb} - 2)$$

This method which has been used by different authors has the disadvantage that the partial solutions are known only for laminar flow conditions and for a limited number of terms in Equation Cb - 2. It is often not possible to express a temperature variation occurring on an object by a power series with this limited number of terms satisfactorily. For this reason another method will be described here which can be applied to any temperature variation and to turbulent as well as laminar flow. It is mainly based on work by Bond (Ref. 110) and Rubesin (Ref. 120). This method starts from a solution of the boundary layer equations for a step-wise varying wall temperature and replaces an arbitrary wall temperature variation by an infinite number of infinitely small steps. An integration procedure of the known solution for one step then gives the answer for the arbitrary varying wall temperature. This derivation obtains the following equation:

$$q(x) = \int_{\xi=0}^{\xi=x} h(\xi, x) \frac{dT_w(\xi)}{d\xi} d\xi + \sum h(\xi_1, x) [T_w(\xi_1^+) - T_w(\xi_1^-)] \quad (\text{Cb} - 3)$$

for the heat flow at the distance  $X$  from the leading edge into a surface. In this equation  $h$  is the heat transfer coefficient which is found at the location  $X$  when the temperature over the surface is at first equal to the recovery temperature and then jumps suddenly at the location  $\xi$  to a certain value which afterwards is again constant.  $T_w$  is the temperature variation actually found along the surface. The integration is to be performed disregarding discontinuities in the wall temperature

$T_w$ . The summation expressed by the second term on the right hand side of the equation has to be considered when the wall temperature  $T_w$  varies discontinuously along the surface. In this case  $T_w(\xi_1^+)$  is the wall temperature immediately downstream of a sudden temperature jump and  $T_w(\xi_1^-)$  is the corresponding temperature immediately upstream of the jump. If the temperature of the surface differs at the leading edge from the recovery temperature then this summation has also to be applied to the leading edge with a temperature  $T(\xi^-)$  equal to the recovery temperature. The procedure will be explained once more for a specific temperature variation later on.

The following expressions for the heat transfer coefficients which are needed in the above equation have been found by approximate calculations for the conditions on a flat plate. For a laminar boundary layer (Ref. 121)

$$h(\xi, x) = \frac{0.33 k}{x} \sqrt{Re} \sqrt{Pr} \left[ 1 - \left( \frac{\xi}{x} \right)^{3/4} \right]^{-1/3} \quad (Cb - 4)$$

for a turbulent boundary layer (Ref. 122)

$$h(\xi, x) = \frac{0.0289 k}{x} Re^{.44} Pr^{.43} \left[ 1 - \left( \frac{\xi}{x} \right)^{.44} \right]^{-1/9} \quad (Cb - 5)$$

These equations have been verified by experiments. Since the calculation procedure described in this chapter is based on more simplifying assumptions than the results discussed for a constant wall temperature, it seems advantageous to determine by the method of this chapter only a correction factor which has to be applied to the constant wall temperature heat transfer coefficients in order to take into account the influence of a wall temperature variation. This calculation will be done in the following lines:

Neglecting the influence of the variable wall temperature on heat transfer by using the heat transfer coefficient for a constant wall temperature the heat flow at the location  $x$  is described by the following equation

$$q_c(x) = h(0, x) [T_w(x) - T_r] \quad (Cb - 6)$$

From this and the previous Equation Cb - 3 we obtain the relation

$$\frac{q(x)}{q_c(x)} = \frac{\int_0^x h(\xi, x) \frac{dT_w}{d\xi} d\xi + \sum h(\xi_1, x) [T_w(\xi_1^+) - T_w(\xi_1^-)]}{h(0, x) [T_w(x) - T_r]} \quad (Cb - 7)$$

for the ratio of the actual heat flow  $q(x)$  to the heat flow  $q_0(x)$  calculated under the simplifying assumption. For a laminar boundary layer on a flat plate, this equation can be transformed by use of Equation Cb - 4 into

$$\frac{q(x)}{q_0(x)} = \frac{\int_0^x \left[1 - \left(\frac{\xi}{x}\right)^{1/2}\right]^{-1/2} \frac{dT_w}{d\xi} d\xi + \sum \left[1 - \left(\frac{\xi_i}{x}\right)^{1/2}\right]^{-1/2} [T_w(\xi_i^+) - T_w(\xi_i^-)]}{T_w(x) - T_r} \quad (\text{Cb} - 8)$$

and for a turbulent boundary layer on a flat plate into the relationship

$$\frac{q(x)}{q_0(x)} = \frac{\int_0^x \left[1 - \left(\frac{\xi}{x}\right)^{1/3}\right]^{-1/3} \frac{dT_w}{d\xi} d\xi + \sum \left[1 - \left(\frac{\xi_i}{x}\right)^{1/3}\right]^{-1/3} [T_w(\xi_i^+) - T_w(\xi_i^-)]}{T_w(x) - T_r} \quad (\text{Cb} - 9)$$

Each of the above two equations can be solved as soon as the wall temperature  $T_w$  as a function of the distance from the leading edge is prescribed, for instance by numerical or graphical integration of the integrals appearing in the equations.

Let us assume, for instance, that the wall temperature as indicated in the upper diagram of Figure 13 is prescribed. This temperature variation has two discontinuities, one at the leading edge ( $\xi = 0$ ) and one at  $\xi = \xi_1$ . The local heat flow at the location  $x$  for laminar flow may have to be determined. In order to find the integral in Equation Cb - 8 build the differential quotient  $dT_w/d\xi$ , multiply this value with  $\left[1 - (\xi/x)^{1/2}\right]^{-1/2}$  and plot the produce over  $\xi$  as indicated in the lower diagram of Figure 13. The area under this curve represents the integral. The summation has to be extended over  $\xi_i = 0$  and  $\xi_i = \xi_1$ :

$$\sum \left[1 - \left(\frac{\xi_i}{x}\right)^{1/2}\right]^{-1/2} [T_w(\xi_i^+) - T_w(\xi_i^-)] = [T_w(0^+) - T_r] + \left[1 - \left(\frac{\xi_1}{x}\right)^{1/2}\right]^{-1/2} [T_w(\xi_1^+) - T_w(\xi_1^-)] \quad (\text{Cb} - 10)$$

The calculation is however quite tedious because the integration has to be performed for any location  $x$  along the surface for which the local heat flow is to be determined. Additionally, the terms under the integral signs tend always towards infinity when the variable  $\xi$  under the integral sign approaches the limit  $x$  as shown in Figure 13. Therefore the contribution of the region near  $x$  to the integral has to be evaluated by some special procedure, as suggested in Appendix IV.

The denominator in both Equations Cb - 8 and Cb - 9 can be written also in the following form:

$$\int_0^x \frac{dT_w}{d\xi} d\xi + \sum [T_w(\xi_i^+) - T_w(\xi_i^-)] \quad (\text{Cb} - 11)$$

It is in the example equal to the area under the dashed curve in Figure 13 plus the sum of the two sudden temperature changes at the leading edge and at  $\xi_1 = \xi_2$ .

The procedure described assumed that the wall temperature distribution is known. The inverse problem to find the wall temperature for a prescribed convective heat flow has been solved in Reference 117. Actually, the wall temperature has usually to be determined from a heat balance between the convective heat transfer and the radiative heat exchange of the surface with the surrounding and the heat conducted away into the solid interior. Generally, this problem can be solved only by a tedious trial and error procedure.

In this situation it is important to note that by treatment of some simplified case it has been determined that the ratio of the differences between wall and stream temperatures, calculated by the procedure described in this chapter, to the corresponding temperature difference based on a simplified procedure which neglects the influence of the wall temperature variation on the heat transfer coefficient is considerably smaller than the ratio of the heat transfer coefficients determined by both procedures.

Lighthill (Ref. 119) based his calculations on the assumption that a thin-walled flat plate receives heat by laminar convection from a gas stream and discharges heat by radiation into a surrounding with a temperature of  $0^\circ$  Rankine. Heat conduction within the wall and any other heat removal has been neglected. Using the expressions given by Equations Cb - 3 and Cb - 4 for the convective heat flow into the surface and balancing this heat flow by the radiative heat transfer an integral equation is obtained which has been solved. The result of this calculation which is contained in Figure 2 of the referenced report can be used to make a comparison with a calculation based on heat transfer coefficients for isothermal surfaces. It has been found that the wall temperature calculated by Lighthill differs up to 10 per cent from the absolute wall temperature obtained by the simplified calculations.

Another calculation has been made by Bryson and Edwards (Ref. 112) for the following situation: A flat plate is initially at a constant temperature and is suddenly subjected to the influence of a high velocity gas flow of constant or exponentially decreasing velocity. Under the influence of the convective heat transfer the temperature adjusts itself gradually to the temperature of the stream. Radiation has been neglected and the convective heat flow is balanced with the heat stored within the wall (no heat conduction within the wall parallel to the surface and constant temperature throughout the wall normal to the surface are assumed). The use of Equation Cb - 3 for the heat flow results in an integro-differential equation which has been solved and compared with the results of the approximate procedures which neglects the influence of a temperature variation on the heat transfer coefficients. It was found that the difference between wall temperature and static stream temperature as obtained by the exact calculation is up to 20 per cent larger than the difference obtained by the approximate procedure. A difference of 10 per cent in the wall temperature as found by Lighthill or of 20 per cent in the temperature difference as determined by Bryson corresponds at a wall temperature of  $1500^\circ$  Rankine and a static stream temperature of  $400^\circ$ R. to a temperature difference of around  $200^\circ$ F. and such a difference may for instance, be deciding in the decision whether a certain material can be used or not for the skin of an aircraft. It may also be that the temperature difference on special locations like the leading edge and the trailing edge of a solid wing which is being heated up by aerodynamic heating (as calculated in Reference 116) may be larger than in the above

examples. Near the leading edge the temperatures will be increased and near the trailing edge decreased by the effect discussed in this chapter. In design calculations it will therefore have to be decided from the situation whether the influence of a variable wall temperature has to be included into the calculations or whether the much simpler procedure which neglects this influence is considered as sufficiently accurate.

A check on the accuracy of the method discussed in this chapter has been made by Lighthill who compared it with Chapman and Rubesin's solution for laminar flow and a wall temperature variation according to Equation Cb - 1. Agreement within 3 per cent was obtained.

## SECTION V

### FRICTION AND HEAT TRANSFER AT LOW DENSITIES

The solutions for friction and heat transfer which have been presented up to now are subject to a number of limitations. One of these limitations will be discussed in the present chapter. It is connected with the fact that air consists of single molecules. In deriving the differential equations which are the basis for the relationships developed, it has been assumed that the properties of the fluid vary steadily throughout the field investigated. Actually energy and momentum is carried without change by the single molecules from one to the next collision. As long as the mean free path length of the molecules is very small compared with the length dimensions with which we are concerned, we can assume the variation to occur steadily.

The mean molecular path length increases inversely proportional to the density and therefore with decreasing density it has to be expected that a limit is reached where the free molecular path length is of the same order of magnitude as the characteristic dimension. The ratio of free molecular path length to the characteristic length is called Knudsen number. In boundary layer flow as it has been considered up to now the rate of change of the properties is largest in a direction normal to the surface throughout a length equal to the boundary layer thickness. For this type of flow the Knudsen number is defined as the ratio of mean free molecular path length to boundary layer thickness. The Knudsen number  $Kn$  may be expressed by the flow parameters which have been used before, namely the Reynolds number and the Mach number. From gas kinetic considerations the following relationship is derived

$$Kn = \frac{\sqrt{T}}{0.797} \frac{Ma}{Re\delta} \quad (D - 1)$$

in which  $\gamma$  is the ratio of specific heats at constant pressure and volume.

For boundary layer flow the Reynolds number is based on the boundary layer thickness as characteristic length. Changing to the Reynolds number based on the distance along the surface measured from the leading edge results in the following relationships for laminar flow along a flat plate

$$Kn = \frac{\sqrt{T}}{3.7} \frac{Ma}{\sqrt{Re}} \quad (D - 2)$$

and for turbulent flow

$$Kn = \frac{\sqrt{T}}{0.5} \frac{Ma}{(Re)^{0.8}} \quad (D - 3)$$

According to the above discussion the molecular structure makes itself felt when the Knudsen number exceeds a certain limit. This limit can be well illustrated in a diagram which uses the Reynolds number and the Mach number as abscissa and ordinata respectively. Such a diagram is presented in Figure 14. Inserted in the diagram is the approximate limit between turbulent and laminar flow. This limit is determined by a large number of factors which will be discussed in more detail in a later chapter. The line given in Figure 14 should be taken only as a very approximate indication. The line with the break on this boundary between laminar and turbulent flow indicates the conditions for which the Knudsen number for laminar flow as an expression for the ratio of free molecular path length to boundary layer thickness has the value 0.003. In the field on the right hand side of this line the free molecular path length is less than 1 per cent of the boundary layer thickness and the relationships developed in the preceding chapters apply.

With increasing distance from the limiting line towards the left hand side the free molecular path length increases more and more and it has to be expected that the molecular structure of the air causes larger and larger deviations from the conditions encountered in a continuum. Near the limiting line these deviations occur only within the immediate neighborhood of the solid surface. Their main effect is to create finite flow velocities right at the wall surface and a sudden variation of the temperature (temperature jump). Accordingly the regime just to the left of the limiting line is called slip flow regime.

With decreasing density finally a state is reached where now inversely the free molecular path length is much larger than any dimension of the body. In such a case the molecules impinge on the body surface practically with the momentum and energy which prevail in large distance from the body. The field in Figure 14 in which this condition applies is termed free molecular flow regime. The limit for this regime is usually fixed by the condition that the value  $Ma/Re$  which is not much different from the Knudsen number has the value 10 (Ref. 140)

$$Ma / Re = 10$$

(D - 4)

No boundary layer exists under this condition. Therefore  $Re$  is based on a characteristic body dimension. The line described by Equation D - 4 is also shown on the left hand side of Figure 14.

Another limit to the relationship for continuum flow presented in the previous chapters is connected with the fact that with decreasing Reynolds number and increasing Mach number the boundary layer thickness increases. The boundary layer equations on which the solutions are based become invalid when the boundary layer thickness exceeds a certain limit. The displacement of the flow by the boundary layer becomes then so large that it causes the pressure to vary notably along a flat plate. This effect is especially large where the flow field is limited by a shock wave and when this shock wave is comparatively near to the surface. Different authors have made estimates on the limit beyond which this boundary layer shock wave interaction becomes important. The results indicate generally that the limit is situated near to the limit between the slip flow and continuum flow region somewhat displaced toward the left for low Mach numbers and toward the right at high Mach numbers. Correspondingly, for high Mach numbers the slip effect will be the first one which modifies the continuum flow conditions when the Reynolds number decreases, whereas at low Mach numbers the mentioned boundary layer shock wave

interaction will cause the first modification.

In general the friction and heat transfer conditions outside the range of continuum flow are extremely complicated and therefore little investigated analytically as well as experimentally. A comparatively good knowledge exists only for the regime far to the left in Figure 14 which has been termed "free molecular flow" regime. Very much work remains to be done in the "slip flow" regime and especially in the "mixed flow" regime.

In this situation it is important to visualize the regions in the Reynolds-Mach number field which will be of special importance in future aeronautical applications. In this paper we are concerned with cooling problems and from this point of view we can obtain the desired information in the following way: Cooling problems will always become critical when the skin of the aircraft exceeds certain temperature limits. The surface temperature of the aircraft is determined by the combined effects of convective heat transfer from the atmosphere to the surface, of emission of radiation from the surface, of irradiation from the sun, and finally of cooling either by storage of heat in the solid material of the aircraft or by some cooling process. All of these heat exchange processes depend on the flight altitude and the flight velocity, both of which can be expressed by the Reynolds number and Mach number at which the flight occurs. Two curves calculated on the basis of the mentioned considerations in Appendix II have been inserted in Figure 14. Along these curves a wall temperature of  $1500^{\circ}\text{R}$  is obtained by the surface of the aircraft when the surface radiates heat like a black body and when cooling by heat storage or by some cooling process does not occur. One of the curves holds for a location on the aircraft which has a distance of one foot from the leading edge and the second curve for a distance of 5 ft. The break in the curve at  $\text{Re} = 2 \cdot 10^6$  is connected with the fact that cooling conditions are more favorable in a laminar boundary layer than in a turbulent one. Depending on the specific conditions the location of the transition point may shift to the right or left from the position indicated in the figure.

Reynolds number and Mach number together with the other data which have been mentioned, fix the flight altitude along the two curves and the corresponding values are also indicated. The left hand curve holds also for the condition that the distance  $L$  is equal to 5 ft. but the emissivity of the surface for heat radiation is equal to 0.2. The two curves therefore indicate the Reynold and Mach number conditions for which an uncooled surface of an aircraft assumes a temperature of  $1500^{\circ}\text{R}$  in steady flight. If cooling is applied to the surface either by heat storage which is possible for short flight duration or by some cooling process then the flight velocity and correspondingly the Mach number can be higher for a certain Reynolds number without increasing the surface temperature above the value of  $1500^{\circ}\text{R}$ . For this condition therefore the flight would occur at a point which is located in Figure 14 above and to the right of the curve. Aircraft or missile during their flight may encounter conditions as given by their Reynolds and Mach number which belong to the range located below and to the left of the two curves in Figure 14. This then indicates that the wall temperature even for steady flight will be lower than the assumed limit of  $1500^{\circ}\text{R}$  and generally that such flight conditions are not critical from a cooling standpoint. The conclusion therefore is that the range near the two curves in Figure 14 is the one for which information on heat transfer is most important. With this in view we will now review the information on friction and heat transfer which is available for the regimes outside the continuum flow region.



Comparatively good information is available for the conditions in the free molecular flow regime. The kinetic theory of gases gives the possibility to calculate friction and heat transfer to objects in a flow with uniform velocity and uniform temperature. Figure 15 and 16 show such data. The figures have been taken from Reference 135. Figure 15 gives Stanton numbers for a monatomic and a diatomic gas. The information on a monatomic gas is important because the air molecules dissociate at very high altitudes. The value  $\phi$  by which the Stanton numbers are divided in the figure is the accommodation coefficient. Its value for air is not too well known. Usually it is assumed to be equal to 0.9. An interesting fact may be observed in Figure 16 which presents the recovery factors for two-molecular and one-molecular gases as a function of the Mach number. It can be seen that quite a number of configurations among them the flat plate in a flow parallel to its surface have recovery factors which are larger than one. We have therefore to expect that with decreasing density the recovery factor which in continuum flow of air has a value of 0.85 or 0.9 increases and assumes values above 1 in the free molecular range. This trend has actually been verified experimentally.

Information on heat transfer in the mixed flow regime is very limited. This is an unfortunate fact since we have seen that this region is of special importance for future aeronautical applications. Calculations which have been made rest on a comparatively unsecure basis since it is today even not known exactly what the differential equations are which describe the flow and energy transfer process in this region. Experimental information has been obtained at the Ames Laboratory of the NACA and at the University of California in Berkeley. Such information is very valuable in view of the difficulties of obtaining analytic information. However, these tests are restricted to Mach numbers below 4 and from Figure 14 it can be seen that the very high Mach number condition is especially important in this range. It is not at all sure whether information obtained at small Mach numbers can be used for the high Mach number regime as well. One additional factor may be observed in Figure 14. Any information obtained from analysis and all the experimental information obtained so far deal with laminar flow region. The curves indicating the flight conditions under which the surface of an aircraft assumes a temperature of 1500°R however, indicate that at least part of the flight of aircraft and missiles has to be expected to be in the turbulent region, at least near the slip flow regime and there exists practically no knowledge on heat transfer under turbulent flow conditions outside of the continuum regime.

In the rest of this chapter information will be discussed which has resulted from investigations in the slip flow and the mixed flow regime for laminar flow conditions. Near the continuum regime the normal boundary layer equations still hold to a good approximation. Only the boundary conditions have to be changed to take into account the finite velocities and the sudden change in temperature occurring on a solid surface (slip flow and temperature jump) and the interaction between shock waves and boundary layer. Maslen (Ref. 132) has made a calculation on this basis which is essentially a perturbation of the normal continuum solution. This calculation determines to a first approximation the changes which occur in the normal continuum solutions when the flow enters the slip flow regime. These results of the calculations are expressed by the following equations for the skin friction coefficient

$$c_f \sqrt{Re} = (c_f \sqrt{Re})_0 + \epsilon (c_f \sqrt{Re})_1$$

(D - 5)

and for the heat transfer coefficient

$$\frac{Nu}{\sqrt{Re}} = \left(\frac{Nu}{\sqrt{Re}}\right)_0 + \epsilon \left(\frac{Nu}{\sqrt{Re}}\right)_1$$

(D - 6)

In both equations the subscript zero indicates the normal continuum solution, for instance, as discussed in the previous chapters of this paper. The subscript 1 indicates the change which this continuum solution undergoes to a first approximation and the parameter  $\epsilon$  is essentially the Knudsen number:

$$\epsilon = \sqrt{\gamma} \frac{Ma}{\sqrt{Re}}$$

(D - 7)

The calculation procedure is restricted to the assumption of a constant specific heat for the air but is applicable to any variation of viscosity and Prandtl number with temperature. Table II gives the results of a numerical evaluation. The friction factors and heat transfer coefficients as calculated by Young and Janssen (Ref. 59) are used as the continuum solution and the changes occurring to a first approximation have been calculated by the procedure described in Reference 132. From Table II and from the Equations D - 5 to D - 7 it may be observed that the shearing stress increases above the value in continuum flow when the slip flow region is entered. This increase is caused by the boundary layer shock wave interaction. The heat transfer parameter decreases below the value in continuum flow as a consequence of the temperature jump occurring at the surface under slip flow conditions. Both changes are of the order of magnitude of 1 per cent at the limit which is indicated in Figure 14 between continuum flow and slip flow regime. The difference between the actual and the continuum solution increases with decreasing Knudsen number and is about 10 per cent when the parameter  $Ma/\sqrt{Re}$  assumes the value 0.1 indicating that the free molecular path length has increased to approximately 1/10 of the boundary layer thickness.

An indication of how friction and heat transfer change throughout the whole region between continuum and free molecular flow may be obtained from Figures 17 and 18 which are taken from a paper by Shaef (Ref. 138) summarizing the work of the Berkeley group. The figures contain as dashed lines the solutions obtained from the boundary layer equations and valid in the continuum flow regime as well as the solutions obtained for the free molecular flow regime. The full line gives the recommended correlation. Some experimental points have also been included. The Reynolds and Mach numbers appearing in Figure 18 are based on the conditions behind the shock wave. The value  $\theta$  is half the apex angle of the cone.

## SECTION VI

### FRICTION AND HEAT TRANSFER UNDER HIGH TEMPERATURE CONDITIONS

In the flight of aircraft through the atmosphere with extreme Mach number very high temperatures are encountered within the boundary layer. These temperatures cause the air to dissociate and finally to ionize (at temperature above  $20,000^{\circ}\text{R}$ ). In this chapter the limitations will be discussed which are imposed on the boundary layer solutions in the previous chapters under high temperature conditions.

Figure 19 has been prepared to indicate what temperatures have to be expected within the boundary layer at high Mach numbers. The figure holds for a stream static temperature of  $400^{\circ}\text{R}$ . When the surface assumes the recovery temperature then this temperature will also be the highest static temperature which is encountered within the boundary layer. This recovery temperature of a flat plate is indicated in the figure for laminar and turbulent flow conditions. The full line has been calculated assuming constant specific heat and a recovery factor of 0.85 for laminar flow. The dashed line is calculated for a recovery factor of 0.9 corresponding to turbulent flow and again assuming constant specific heat. A third curve has been added from the results of Reference 134, which considered the specific heat to vary, in order to indicate what variation in the recovery temperature is caused by the variation of the specific heat.

In aircraft applications the surface temperature will usually be cooled by radiation or by some cooling process. Under such conditions the maximum temperature occurs somewhere within the boundary layer (See also Fig. 5). Curves have been added in the figure which indicate this maximum temperature within the boundary layer for the case that the wall temperature is cooled down to the stream static temperature and for the case that the absolute wall temperature is four times the free static temperature (at a temperature of  $1600^{\circ}\text{R}$ ). For a gas with a Prandtl number equal to 1 it can be easily derived that the difference between the maximum temperature within the boundary layer and the stream static temperature is equal to one fourth of the difference between recovery temperature and static temperature in the stream for laminar as well as for turbulent flow under the condition that the wall temperature is equal to the static stream temperature. For the calculation of the curves in Figure 19 it has been assumed that this condition still holds approximately for air with a Prandtl number of 0.72.

It can be determined from Reference 165 that dissociation starts at an altitude of 50 miles in the air with a temperature around  $3,000^{\circ}\text{R}$ . and at sea level with a temperature around  $6,000^{\circ}\text{R}$ . The Figure 19 in which these limits are also inserted indicates that no dissociation within the boundary layer is expected up to Mach numbers of 6 or 10 for the case where the wall temperature is kept at the recovery temperature and that dissociation will occur only at Mach numbers above 12 to 18 when the wall temperature is cooled to values below  $1600^{\circ}\text{R}$ . The latter case is the one which is more likely to occur on aircraft.

Two papers (Ref. 142, and 143) have been published recently which investigate what effect dissociation will have on friction and heat transfer in laminar flow along a flat plate. Such calculations are connected with considerable uncertainties. They are for instance based on the assumption that the air adjusts itself at each location within the boundary layer to the equilibrium dissociation belonging to the local temperature. It is actually quite doubtful whether the fluid particles stay

for a sufficient time in the high temperature zone to assume equilibrium dissociation. Also diffusion of the different gases into which the air dissociates into the cooler regions may occur. All these facts are neglected in the calculations. Another uncertainty is based on our insufficient knowledge of property values of a gas at extremely high temperature and under the condition of dissociation. In this point the assumptions made in both sets of calculations differ considerably. For instance Moore (Ref. 143) assumes that the Prandtl number in dissociated air increases up to values around 6 whereas Crown (Ref. 142) reasons that even with dissociation the Prandtl number can never assume values larger than 1. Nevertheless both calculations come in some important points to the same conclusion. It is found that dissociation under the assumed conditions has a negligible effect on the skin friction and on the heat flow entering or leaving the surface per unit area as long as the surface temperature is kept at values which are below the temperature at which the dissociation in the air starts. This situation holds even though very much higher temperatures may be encountered within the boundary layer. The condition that the wall temperature is lower than the dissociation temperature--lower than say 3,000°R--will be fulfilled in almost all aeronautical applications.

Crown finds additionally in his calculations which are based on the assumption that the Prandtl number is constant and that the product of viscosity and density is a linear function of the enthalpy that the recovery factor can be represented within a few per cent by the well-known relationship

$$r_1 = \sqrt{Pr} \quad (E - 1)$$

He also finds in Equation 58 of his report an expression which can be transformed into the following relation for the ratio of Stanton numbers to friction factor

$$2 \frac{St}{c_f} = \frac{Pr}{c_{p_s}}^{-2/3} = \frac{i_r - i_w}{T_r - T_w} \quad (E - 2)$$

This equation is identical with Equation Ba - 8 and indicates that the Stanton number is practically independent of dissociation when based on enthalpy.

Moore performed his calculation considering that the Prandtl number for air varies with temperature. Enthalpy recovery factors calculated from the recovery temperatures contained in his report have been compared with enthalpy recovery factors calculated from equation

$$r_1 = \sqrt{Pr^*} \quad (E - 3)$$

with the Prandtl number introduced at the reference enthalpy as given by Equation Ba - 26. The agreement is not too good for this set of data. Formula (E - 3) gives recovery temperatures which are up to 20 per cent too high. For design

calculations the actual recovery temperature under conditions of dissociation is of little interest since it is so high that no presently known material can withstand it. Surfaces of aircraft flying at such Mach numbers will have to be cooled considerably. Into calculations of the convective heat transfer under such conditions the recovery temperature enters only as a means to express the heat flow conveniently (Equation Ba - 3) and for this purpose the recovery temperature determined by Equation E - 3 gives better results than the actual recovery temperature.

Crown presented in his paper formulas for a calculation of friction and heat transfer which are valid for any temperature dependence of the properties in the same way as the equations presented in the preceding chapters. The method discussed in this paper is believed to have the advantage that it is in accord with the procedures with which in heat transfer calculations a variation of properties is accounted for.

The findings which have been discussed are very useful information for an estimate of heat transfer coefficients under high temperature conditions. Of course, it has to be kept in mind that they are based on assumptions which still have to be backed up by experiments and that they hold directly only for laminar boundary layers. Experimental investigations under high temperature conditions are needed for more accurate knowledge of friction and heat transfer conditions. They are however, extremely difficult to perform. In wind tunnels for instance, it is almost hopeless to increase the temperatures to such values that on surfaces, which are cooled to conditions similar to the ones expected on high velocity aircraft the influence of dissociation can be studied. From Figure 19 for example, it can easily be determined that for a static temperature of  $400^{\circ}\text{R}$ . in an air stream and a pressure corresponding to an altitude of 50 miles the total temperature or stagnation temperature has to be raised to values above 8,000 to 10,000 $^{\circ}\text{R}$ . before any dissociation will occur in boundary layers which develop on surfaces cooled to a temperature of 1600 $^{\circ}\text{R}$ . It is to be expected that even considerably higher total temperatures are necessary to obtain a marked effect of dissociation on friction and heat transfer at the surface.

## SECTION VII

### TRANSITION FROM LAMINAR TO TURBULENT FLOW

In order to apply the information presented in the previous chapters it is necessary to have additional knowledge as to which portion of a surface is covered by turbulent boundary layers or, in other words where the transition from laminar to turbulent flow occurs. This point is the weakest link in the whole procedure to predict wall temperatures and heat transfer coefficients. The present day knowledge has been recently summarized by Gazley in Reference 147. The reader therefore is referred to this reference. Only one figure may be added here which presents the results of a stability calculation by Van Driest (Ref. 145) since it extends to considerably higher Mach numbers than the information contained in Reference 147. Figure 20 shows as a function of Mach number and of wall to stream static temperature the Reynolds number at which boundary layers on a flat plate become unstable for small fluctuations of certain wave length.

The full lines in the figure indicate the critical Reynolds numbers for air of 394°R. static stream temperature assuming a Prandtl number 0.72 and a viscosity which follows Sutherland's relation. Limited by the curve  $Re = \infty$  is a field in which the flow fluctuations always damp out. The border of this field was also calculated for a gas with a viscosity varying proportional to temperature ( $\mu = \text{constant}$ ) and for two Prandtl numbers. The dashed line holds for these conditions. It may be recognized that both parameters have considerable influence on stability.

It is known especially from experiments at a lower Mach number that a certain time or length is necessary for these fluctuations to increase to such an amount that they transform the flow to turbulence. Accordingly the critical Reynolds number for transition is by a factor up to 100 larger than the Reynolds number predicted from Figure 20. However, qualitatively the trends predicted by stability calculations have been verified generally by experiments. The figure can therefore be used to make estimates on the influence of Mach number and wall to stream temperature ratio on the critical Reynolds number at which transition occurs. Even in this respect however, one has to be careful, since Gazley presents experimental information which in some cases does not conform with predictions of the stability theory. Figure 20 together with the information in Reference 147 has been used to estimate the critical Reynolds number for a wall to stream temperature ratio equal to 3 and this estimated curve has been entered into Figure 14. By comparing this curve with the two lines which describe the flight conditions for an aircraft with a surface temperature of 1500°R. it may be concluded that probably up to an altitude around 30 or 40 miles the boundary layers will be turbulent over the major portion of the surface. At higher altitudes, when the aircraft enters the mixed flow region and moves towards the free molecular flow regime, the turbulence has to die down somewhere because certainly no turbulence can be imagined in the free molecular flow regime.

#### ADDENDUM

E. R. Van Driest recently extended his calculations of friction and heat transfer in a laminar boundary layer on a flat plate to cover a very wide range of Mach numbers (Ma 0 to 16) of total stream temperatures ( $T_t$  0 to 2000°F) and of the ratio: enthalpy at surface to enthalpy in stream ( $i_w/i_\infty$  1 to 6) (Ref. 182). Additionally, results for 400°R stream temperature are presented.

The method to calculate friction factors, recovery factors and heat transfer parameters  $2 St/c_p$  with the reference temperature proposed in this report has been checked against the results of Van Driest's calculations. Agreement of the friction factors was within 4 per cent, of the enthalpy recovery factors within 2 1/2 per cent and of the heat transfer parameter within 1 per cent.

## APPENDIX I

### REFERENCE TEMPERATURE

In this appendix it will be described how the Equation Ba - 16 has been obtained from the data in Reference 59. In this paper the reference temperature  $T'$  is tabulated. This temperature is determined in such a way that the friction factor, calculated from the boundary layer solutions and based on property values at the Temperature  $T'$  agrees with the friction factor for a constant property value field.

The temperature  $T'$  will now be approximated by a temperature  $T^*$  as gives by an equation of the form:

$$T^* = T_s + A(T_w - T_s) + B(T_r - T_s) \quad (I - 1)$$

Tables 2, 3, and 4 in Reference 59 represent the temperature  $T'$  for the condition that the wall temperature is equal to the recovery temperature. In this case Equation I - 1 changes to:

$$T^*/T_s = 1 + (A + B) (T_r/T_s - 1) \quad (I - 2)$$

This equation is represented by a straight line in Figure 21. In this diagram the temperature ratio  $T'/T_s$  taken from Reference 59 is plotted over  $T_r/T_s$ . It can be observed that the points representing the exact values of the reference temperature can be well approximated by a straight line. The tangent of the line yields:  $A + B = 0.72$ .

Tables 5, 6, 7 in Reference 59 contain the reference temperature  $T'$  for conditions where the wall temperature is different from the recovery temperature. In order to obtain the constants A and B individually from these data, Equation I - 1 is brought into the form:

$$T^* - T_s = A(T_w - T_s) + (0.72 - A) (T_r - T_s) \quad (I - 3)$$

or:

$$0.72 \frac{T_r}{T_s} - \frac{T^*}{T_s} = -0.28 + A \left( \frac{T_r}{T_s} - \frac{T_w}{T_s} \right) \quad (I - 4)$$

This relation is again represented by a straight line in Figure 22. In this diagram are plotted two values  $0.72 (T_r - T')/T_s$  over  $(T_r - T_w)/T_s$ . These data were calculated from the data in Reference 59. It is seen that these values can also be



well approximated by a straight line. The tangent of the inserted line gives  $A = 0.50$ , which yields:  $B = 0.22$ . In this way Equation Ba - 16 is established.

The same procedure was carried out replacing in all of the above equations the temperature by the enthalpy corresponding to the respective temperature. The points indicated by the white symbols in Figures 21 and 22 were obtained and it may be observed that the same straight line approximates the enthalpy ratios as well as the temperature ratios.

## APPENDIX II

### HEAT BALANCE EQUATION

The temperature which the skin of an aircraft assumes is to be calculated for the following conditions: The aircraft flies horizontally with constant speed for a time sufficient for the skin to assume its steady state temperature. No cooling process is used to remove heat from the skin and the temperature is determined by a balance between the heat exchanged with the universe by radiation. The skin surface may be considered "black" with respect to radiation.

The heat balance per unit area is written:

$$h_1 (i_r - i_w) = \sigma T_w^4 - q_{\text{sun}} \quad (\text{II} - 1)$$

For turbulent flow the Equations Ba-38, Ba-7, and Ba-8 are used to express the heat transfer coefficient by the Reynolds number. The recovery enthalpy is expressed by Equation Ba-21, Ba-47 and the relation:

$$\frac{V_s^2}{2} = \frac{\gamma-1}{2} (Ma)^2 i_s \quad (\text{II} - 2)$$

this results in:

$$0.0296 (Re^*)^{0.8} \{ i_s [1 + 0.18 (Ma)^2] - i_w \} = \frac{Pr^{*1/4}}{\mu} x (\sigma T_w^4 - q_{\text{sun}}) \quad (\text{II} - 3)$$

This equation gives for the prescribed wall temperature of  $T_w = 1500^\circ\text{R}$  and stream temperature the necessary relation between Reynolds - and Mach numbers. The stream temperature, however, is connected with the flight altitude and this value is a function of  $Re$  and  $Ma$  through the following relation for the density of air:

$$\rho^* = \frac{\mu^*}{x a_s} \frac{Re}{Ma} \quad (\text{II} - 4)$$

In addition,  $Pr$  and  $\mu$  depend on the reference enthalpy and therefore on stream temperature (temperature of atmosphere in altitude) and on the Mach number. The problem to determine the Reynolds number which corresponds to a prescribed Mach number may therefore be solved by a trial and error procedure: Assume a stream temperature, calculate from Equation (II - 3) the Reynolds number, from Equation (II - 4) the air density and check in tables for properties of the atmosphere whether the assumed stream temperature corresponds to the altitude as determined by the air density. The result of such calculation for  $x=1$  and  $x=5$  ft is inserted in Figure 14.

Laminar flow conditions occur mainly in the mixed flow regime and the heat transfer coefficients for this region would have to be taken from Figure 18. This, however, makes the calculation very inconvenient. Therefore the calculations were made for laminar continuum flow, expressing the heat transfer coefficient in Equation (II - 1) by Equations (Ba-37) and (Ba-28) and were extended only as far as the continuum relations are expected to hold still with a reasonable accuracy.

The curves of constant wall temperature in the upper right hand corner were calculated with the heat transfer coefficient for free molecular flow.

### APPENDIX III

#### LIMIT FOR ACCELERATION FOR WHICH BOUNDARY LAYER- FLOW CAN BE REGARDED AS QUASI-STEADY

F. Moore shows in Ref. 5 that boundary layer flow along a flat plate may be treated as quasi-steady when the dimensionless parameters:

$$\frac{x V_s'}{V_s^2}, \quad \frac{x^2 V_s''}{V_s^3}, \quad \frac{x^n V_s^{(n)}}{V_s^{n+1}} \quad (\text{III} - 1)$$

are small. In these parameters  $V_s'$  is the first time derivative of the velocity,  $V_s''$  the second etc. The influence of the first one of these parameters on the friction factor and recovery factor of an insulated flat plate with laminar boundary layer has been evaluated. It was found that the shearing stress is larger by 2.5 per cent and the recovery factor smaller by 2.8 per cent than the steady state values for flow with constant acceleration when the condition:

$$\frac{x V_s'}{V_s^2} = 10^{-2} \quad (\text{III} - 2)$$

is fulfilled. This means, for instance, that the acceleration of a plate moving with 1000 ft per sec. speed has to be larger than 10,000 ft/sec<sup>2</sup> or larger than 300 times the gravitational acceleration before deviations larger than 2.5 per cent from steady state conditions are encountered at a location of 1 ft distance from the leading edge. For a distance of 10 ft the necessary acceleration is 1000 ft/sec<sup>2</sup> or 30 g.

# APPENDIX IV

## EVALUATION OF INTEGRALS (Cb - 8) and (Cb - 9) NEAR $\xi_0 = x$

The integrals in Equation (Cb - 8) and (Cb - 9) are of the form:

$$J = \int \left[ 1 - \left( \frac{\xi}{x} \right)^a \right]^b \frac{dT_w}{d\xi} d\xi \quad (\text{IV} - 1)$$

The integral may have been evaluated to such a value  $\xi_0$  near  $x$  that for the remaining interval from  $\xi_0$  to  $x$  it can be replaced by:

$$J = \left( \frac{dT_w}{d\xi} \right)_m \int_{\xi_0}^x \left[ 1 - \left( \frac{\xi}{x} \right)^a \right]^b d\xi \quad (\text{IV} - 2)$$

with the average temperature gradient  $(dT_w/d\xi)_m$  in this interval determinable with sufficient accuracy by an estimate.

By a change of the variable  $\xi$  to:  $y = x - \xi$  and introduction of:  $e = x - \xi_0$  we obtain:

$$J = \left( \frac{dT_w}{d\xi} \right)_m \int_0^e \left[ 1 - \left( 1 - \frac{y}{x} \right)^a \right]^b dy \quad (\text{IV} - 3)$$

Expanding  $(1 - (y/x))^a$  into a series and neglecting terms higher than first order gives:

$$\left( 1 - \frac{y}{x} \right)^a \approx 1 - a \frac{y}{x} \quad (\text{IV} - 4)$$

Introduction into (IV - 3) and integration finally results in:

$$J \approx \left( \frac{dT_w}{d\xi} \right)_m \frac{e^b}{b+1} \frac{e^{b+1}}{x^b} \quad (\text{IV} - 5)$$

The location of  $\xi_0$  can always be chosen such that the approximation (IV - 4) is fulfilled with sufficient accuracy.

Then (IV - 5) gives the contribution of the interval between  $\xi_0$  and  $x$  to the integrals (Cb - 8) and (Cb - 9).

## APPENDIX V

### REFERENCES

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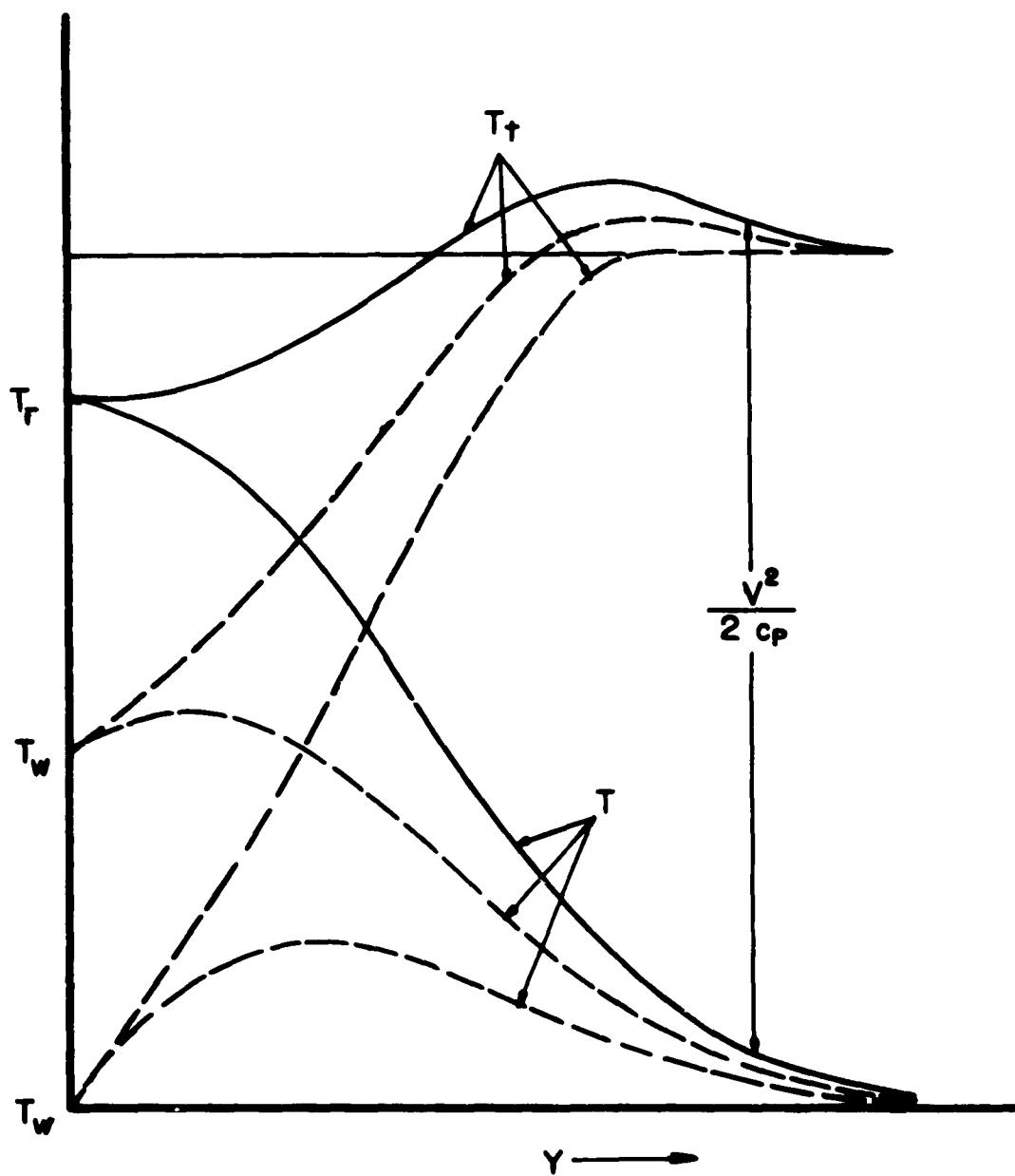


Figure 1. Profiles of Static Temperature  $T$  and Total Temperature  $T_t$  in High Speed Boundary Layer.

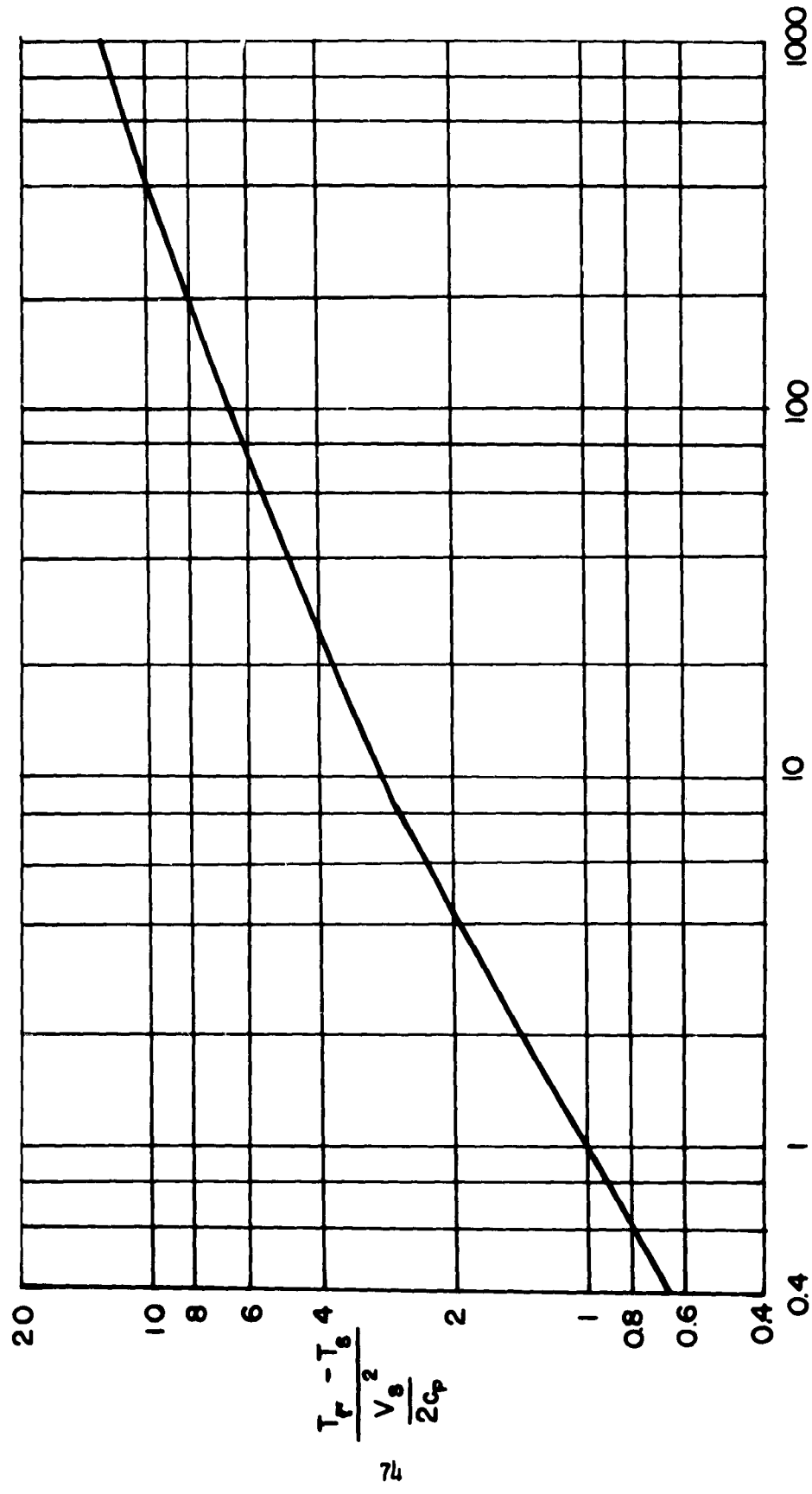


Figure 2. Calculated Laminar Flat Plate Recovery Factor as Function of Pr Number

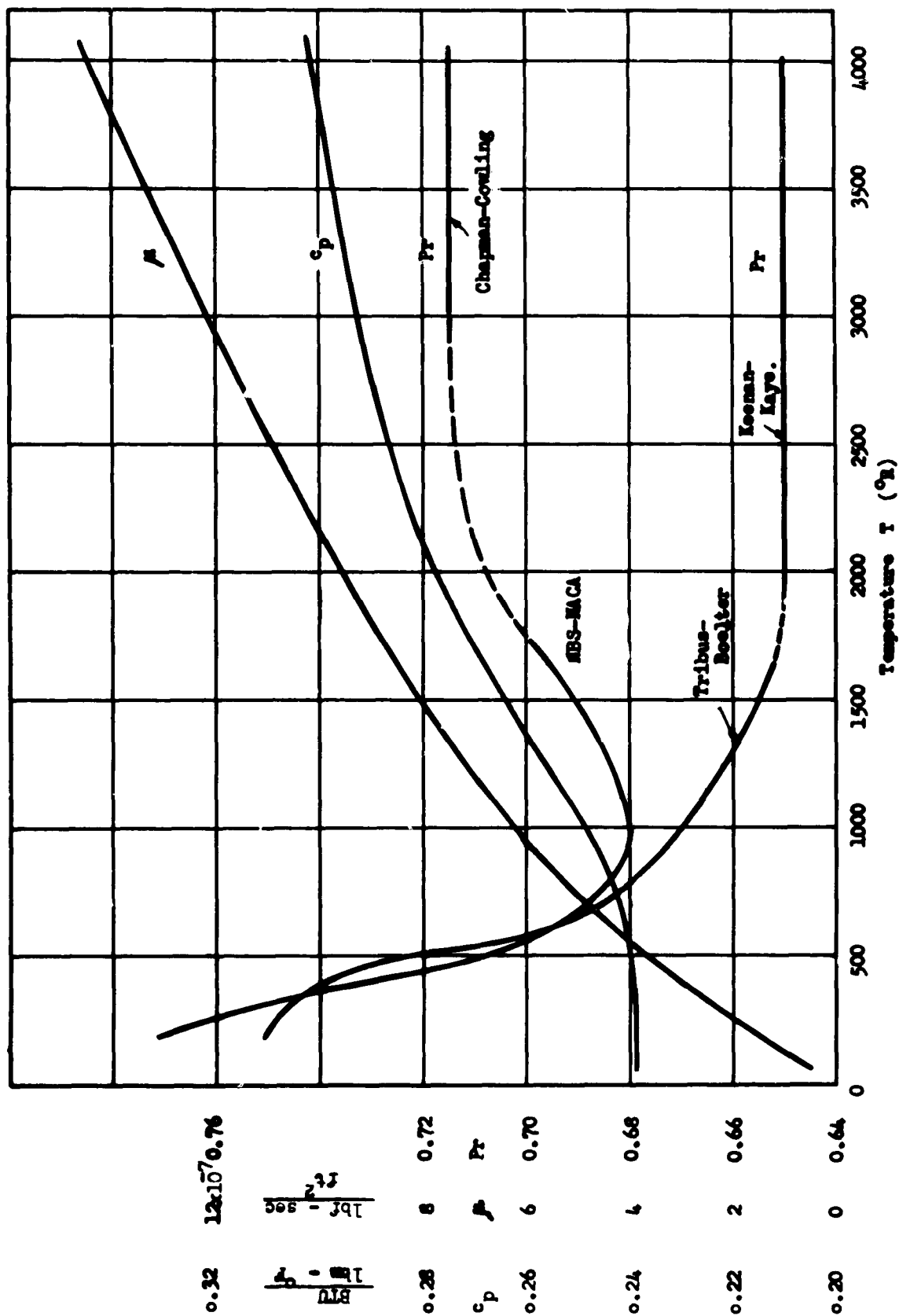


Figure 3. Properties of Air as Function of Temperature.

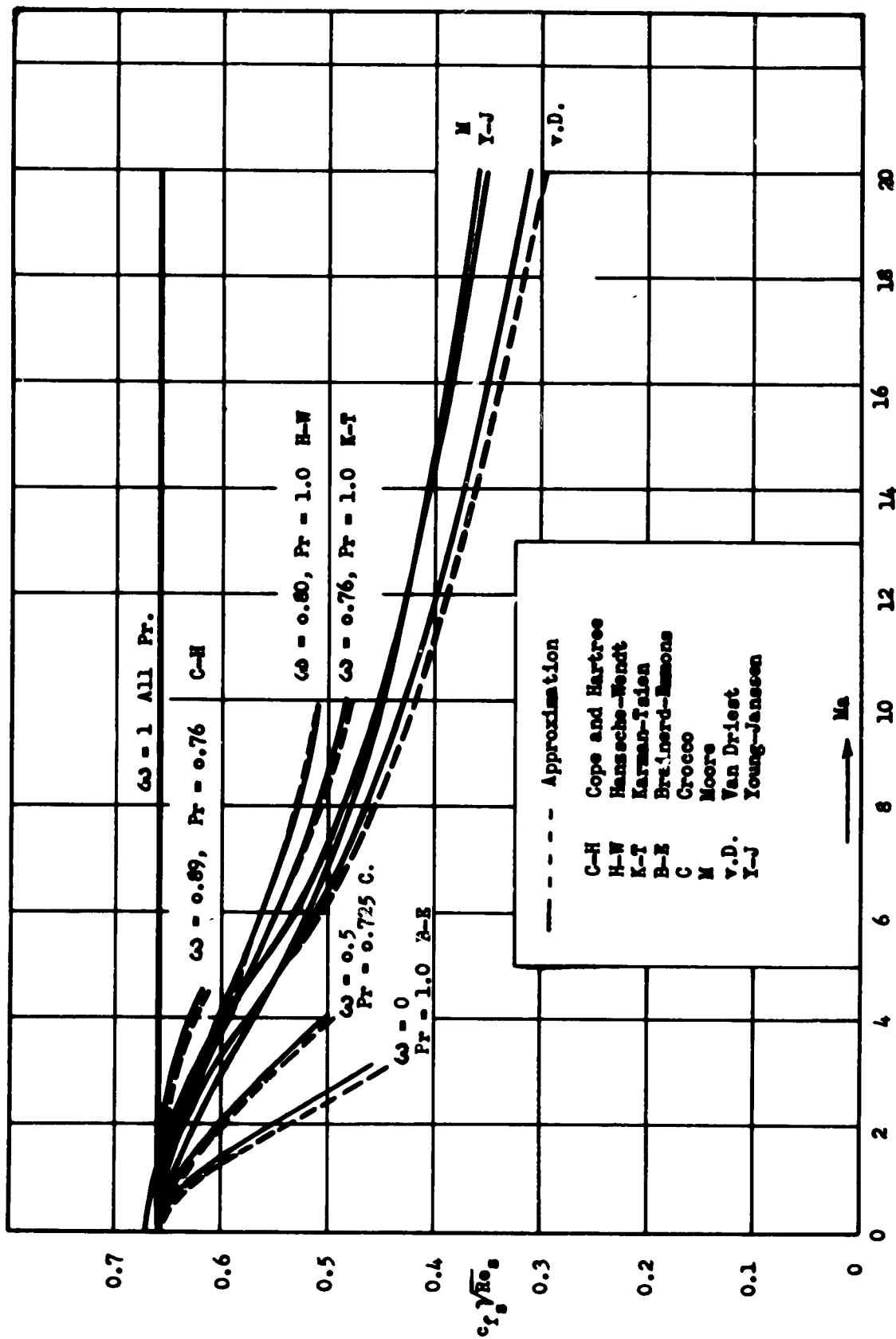


Figure 4. Calculated Skin Friction Coefficient for Laminar Boundary Layers on Flat Plate.

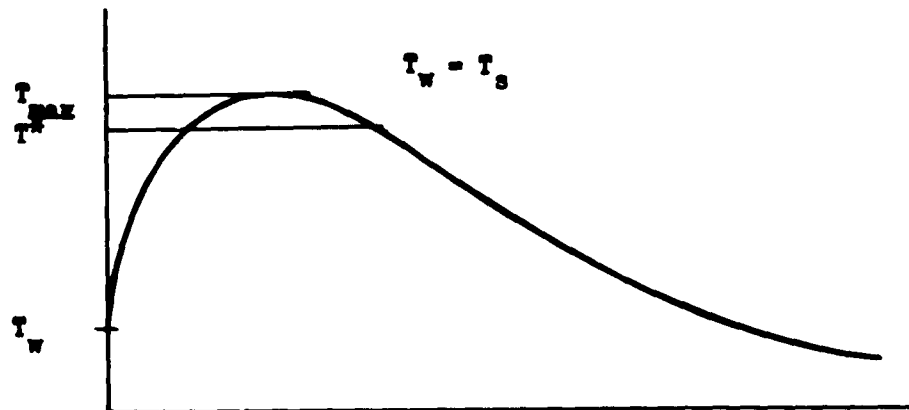
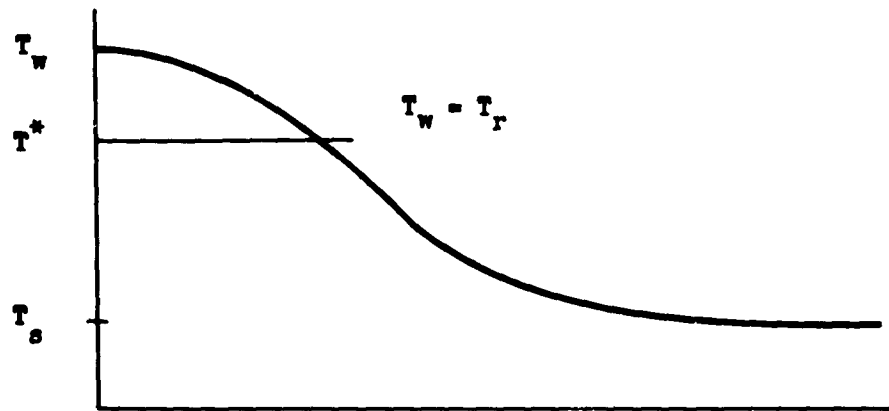
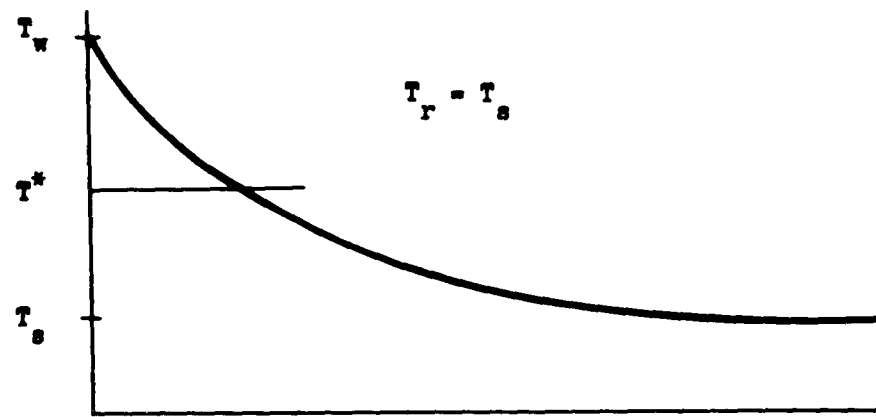


Figure 5. Temperature Profiles and Reference Temperature in High Speed Boundary Layers.

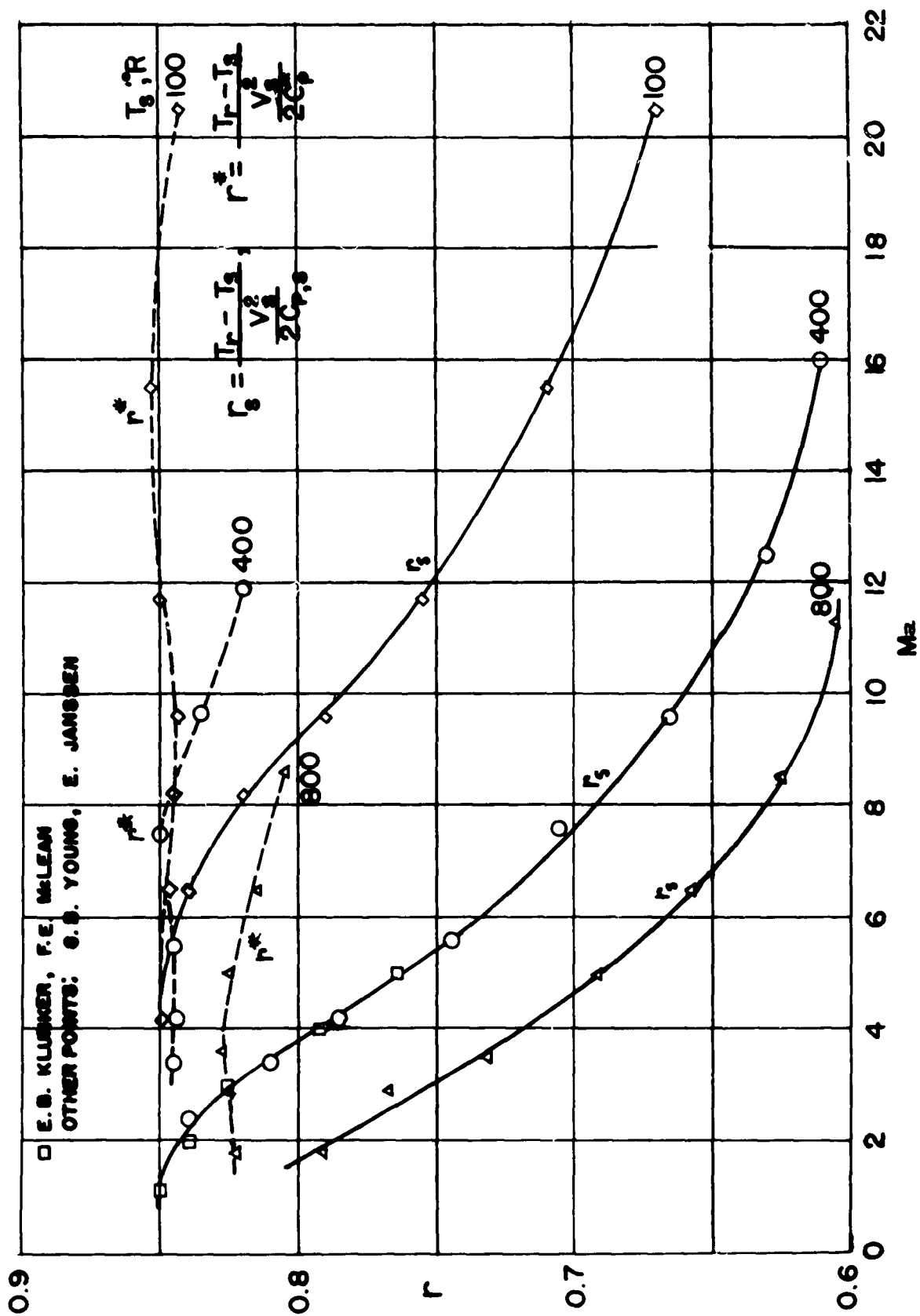


Figure 6. Calculated Laminar Recovery Factors as Function of Mach Number



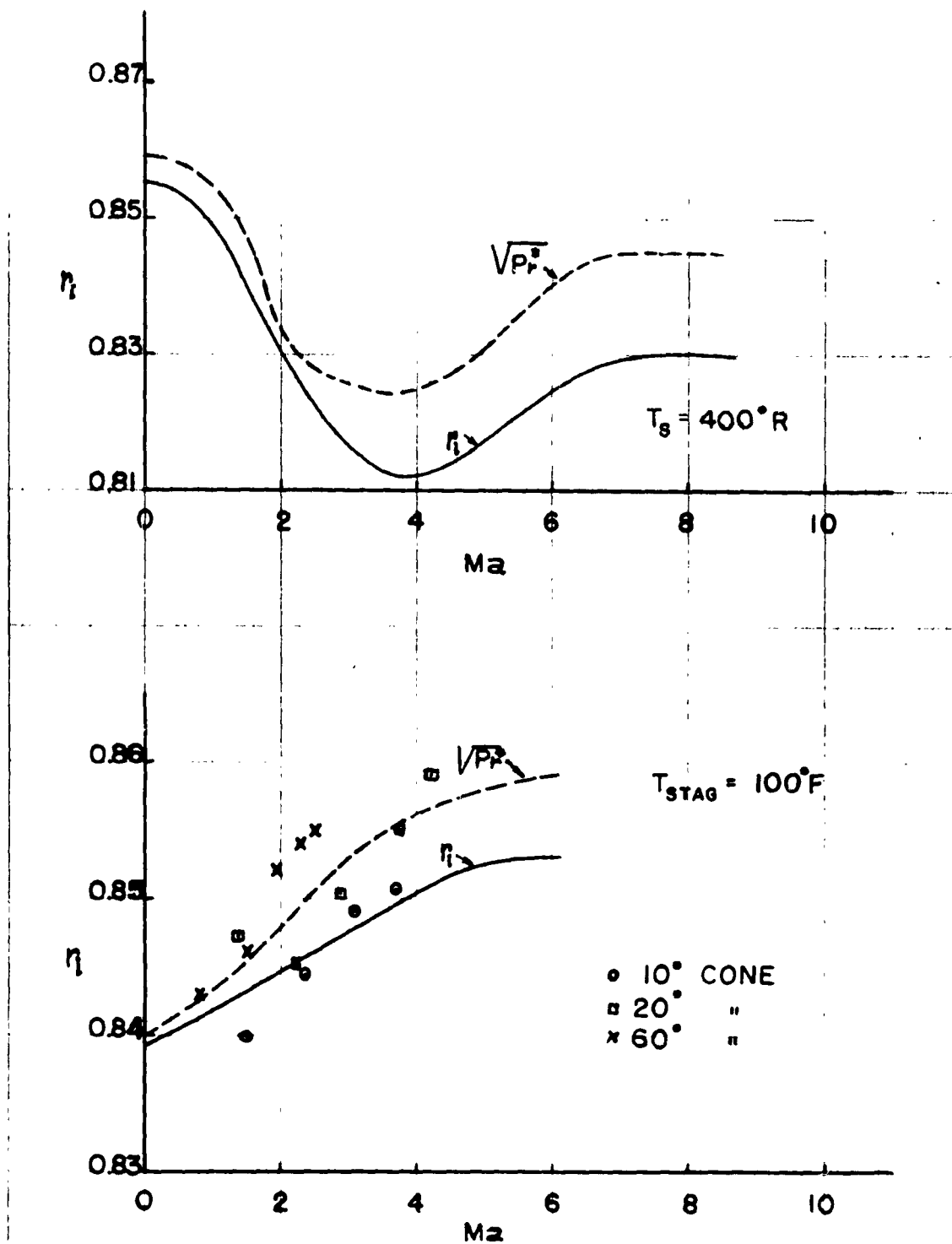


Figure 7. Calculated Laminar Enthalpy Recovery Factors for Flat Plate

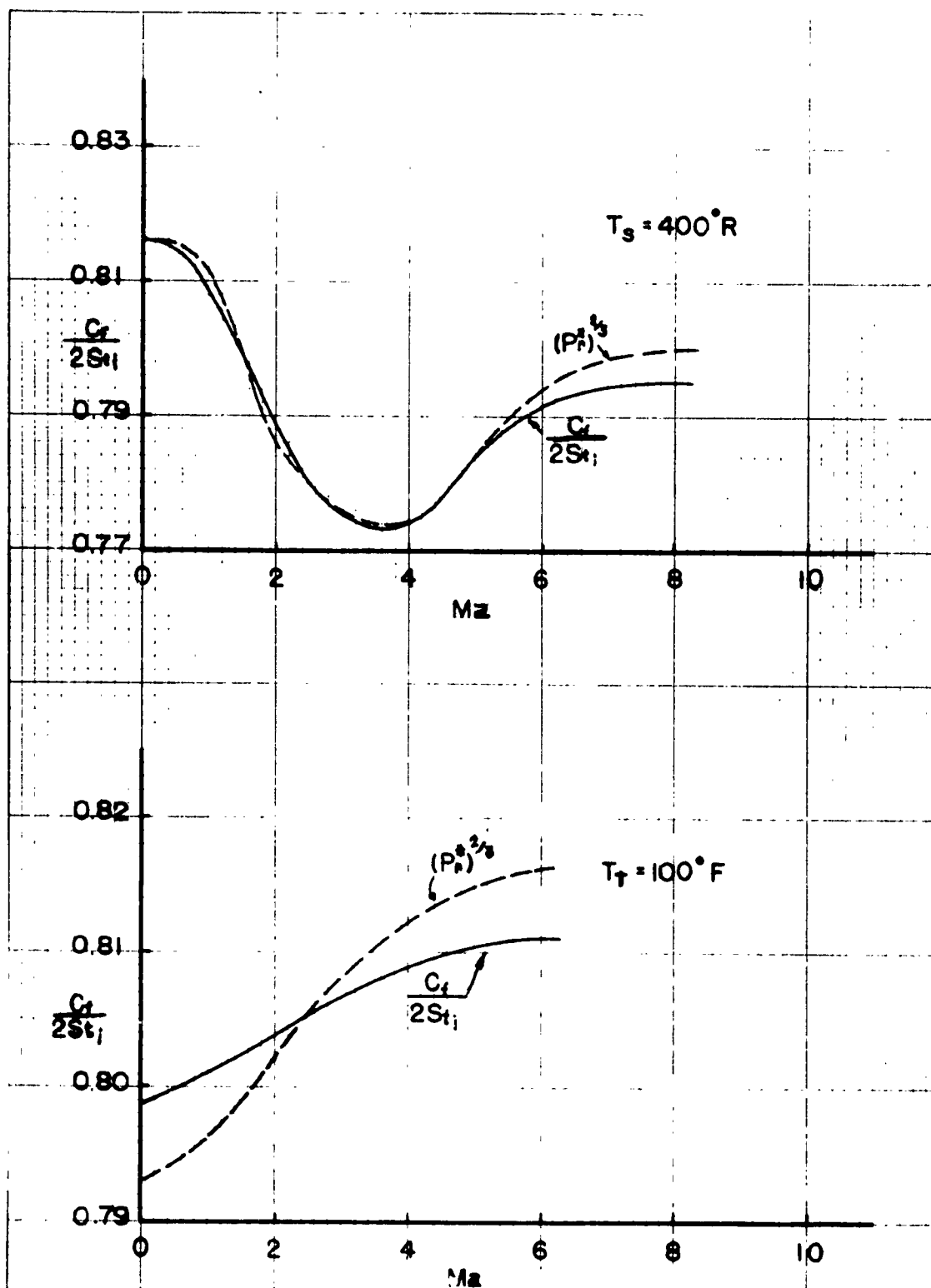


Figure 8. Ratio of Friction Factor to Enthalpy Stanton Number for Laminar Flow over Flat Plate

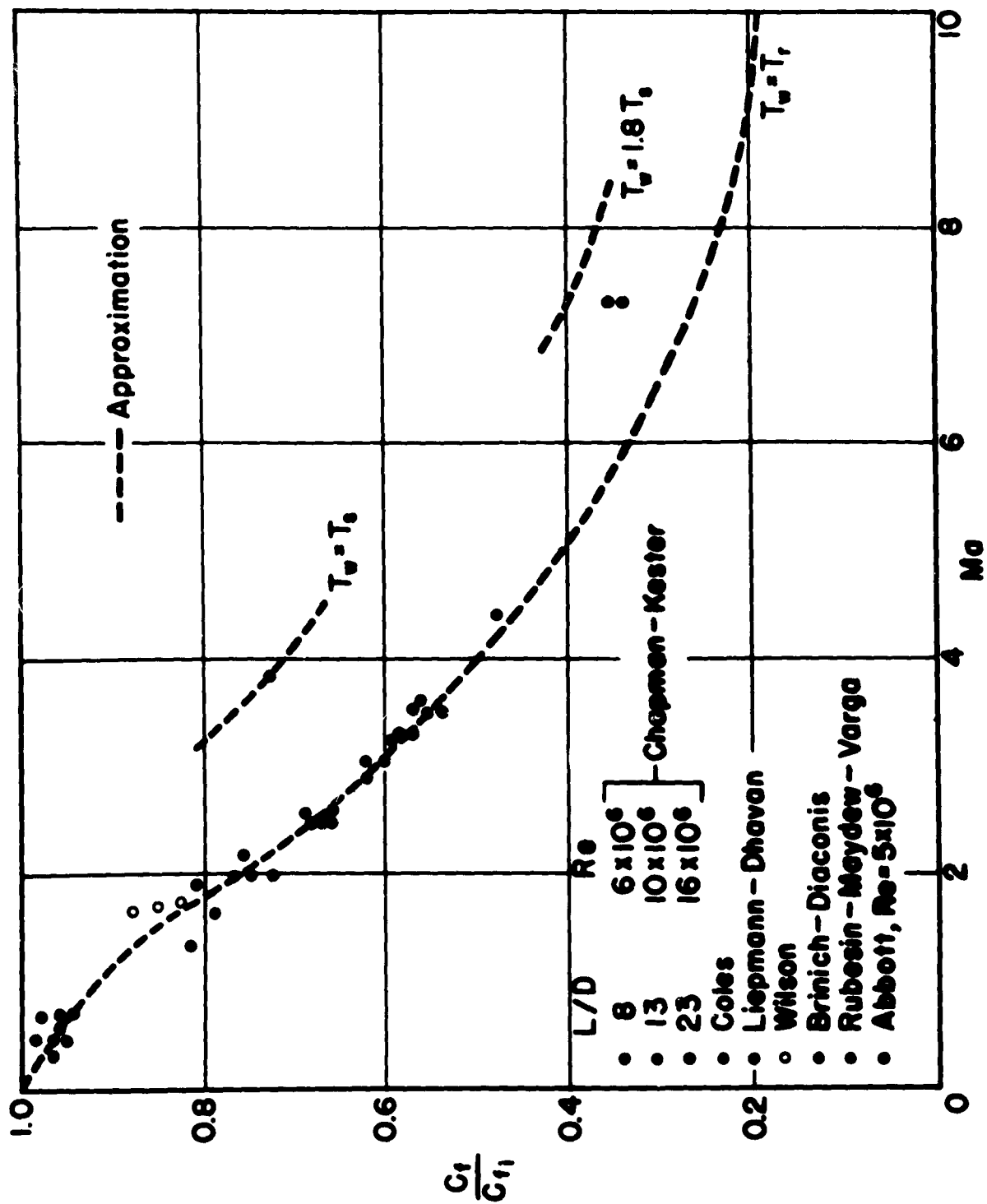


Figure 9. Ratio of Actual to Constant Property Turbulent Friction Factor for Flat Plates

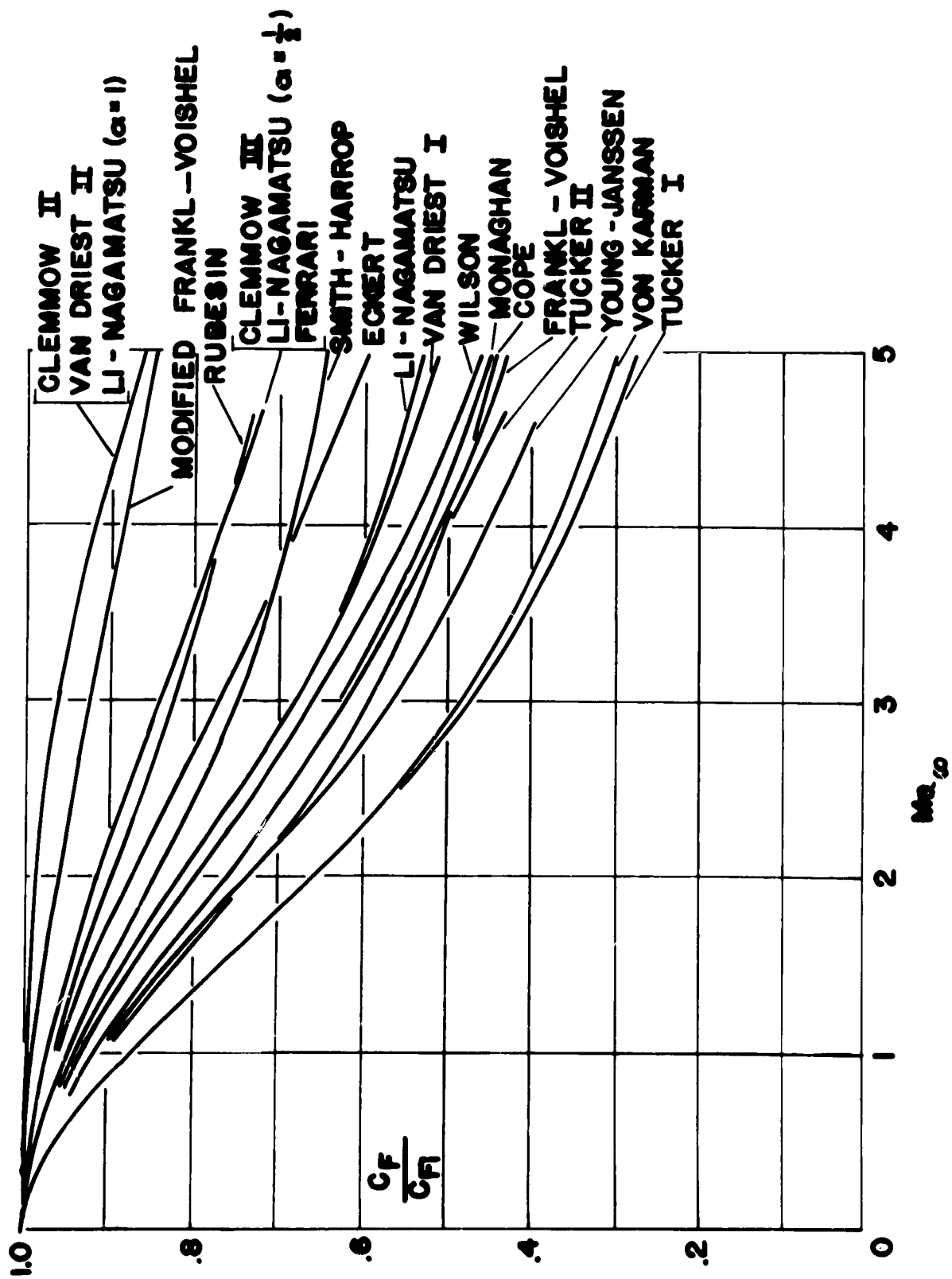


Figure 10. Calculated Turbulent Friction Factors for Flat Plates

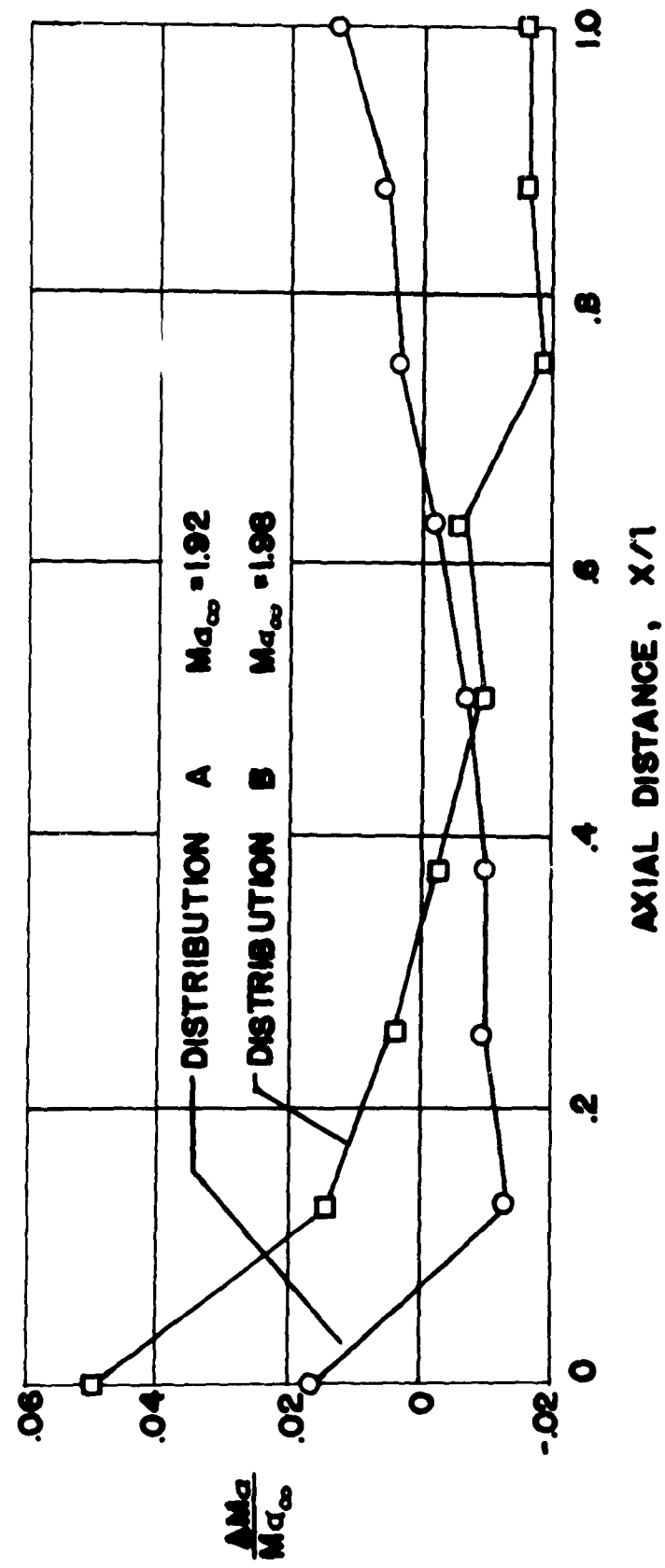


Figure 11. Mach Number Distribution Along Cylinder

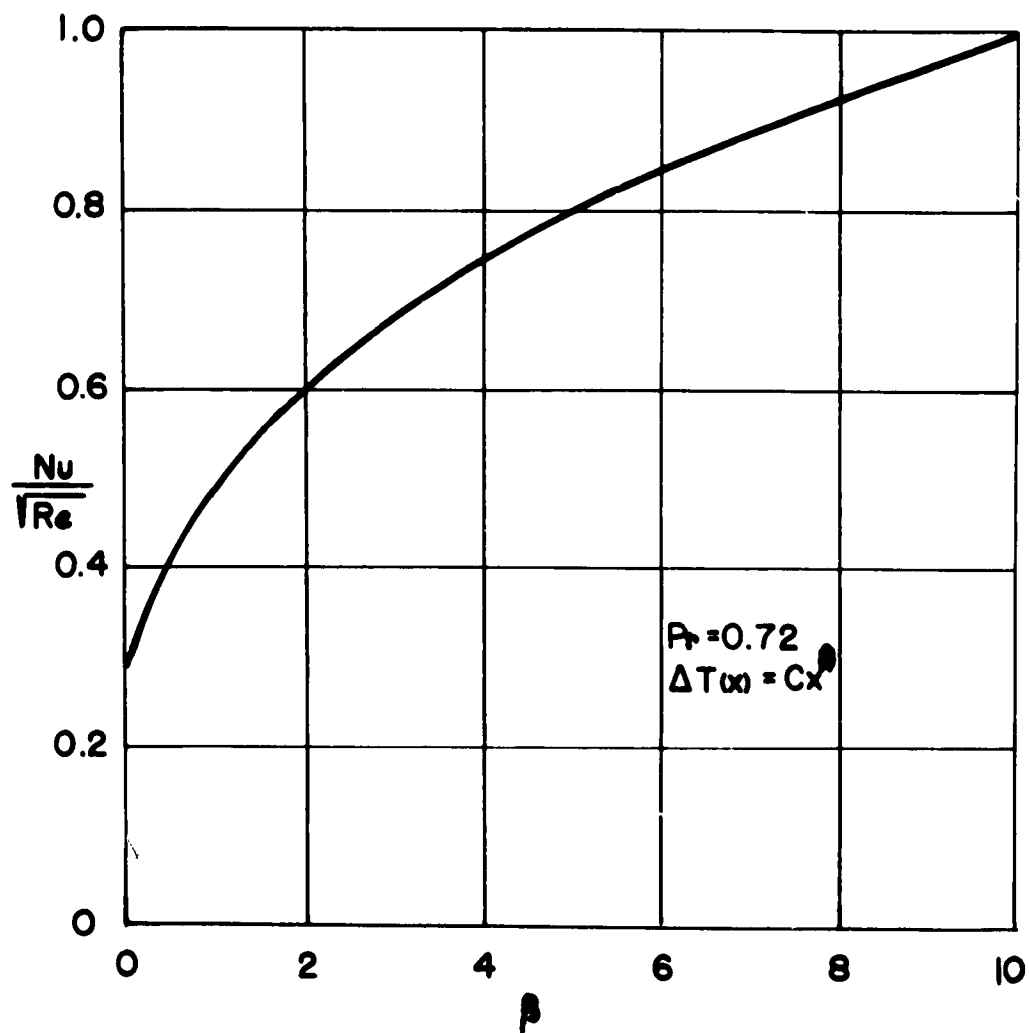


Figure 12. Laminar Heat Transfer on Flat Plate with Variable Wall Temperature.

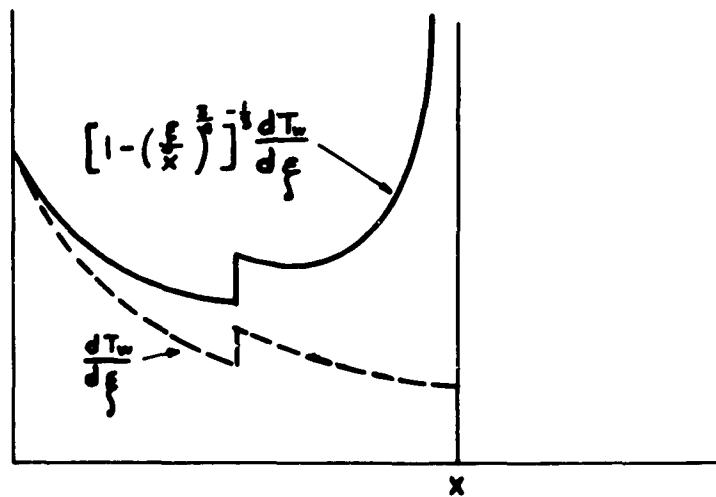
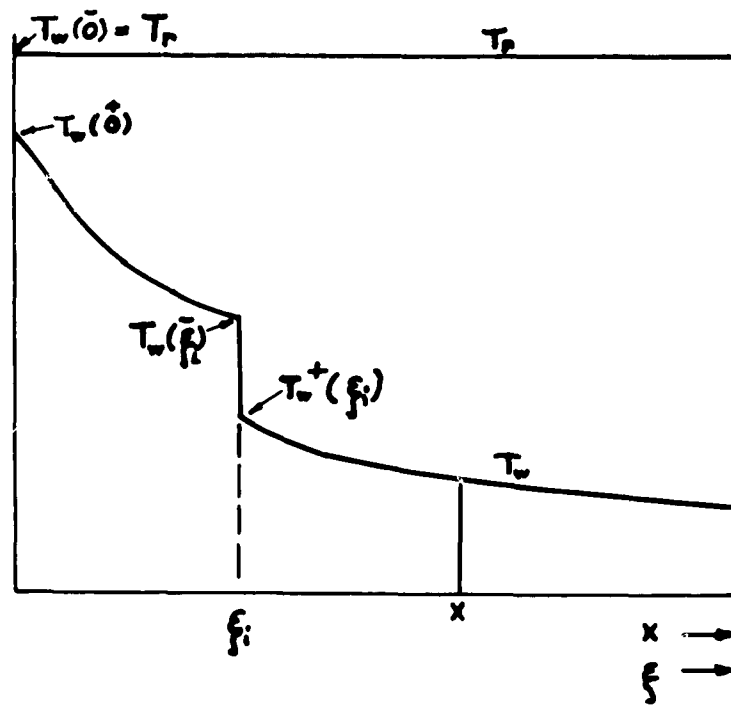


Figure 13. Calculation of Heat Transfer Along Flat Plate with Variable Wall Temperature

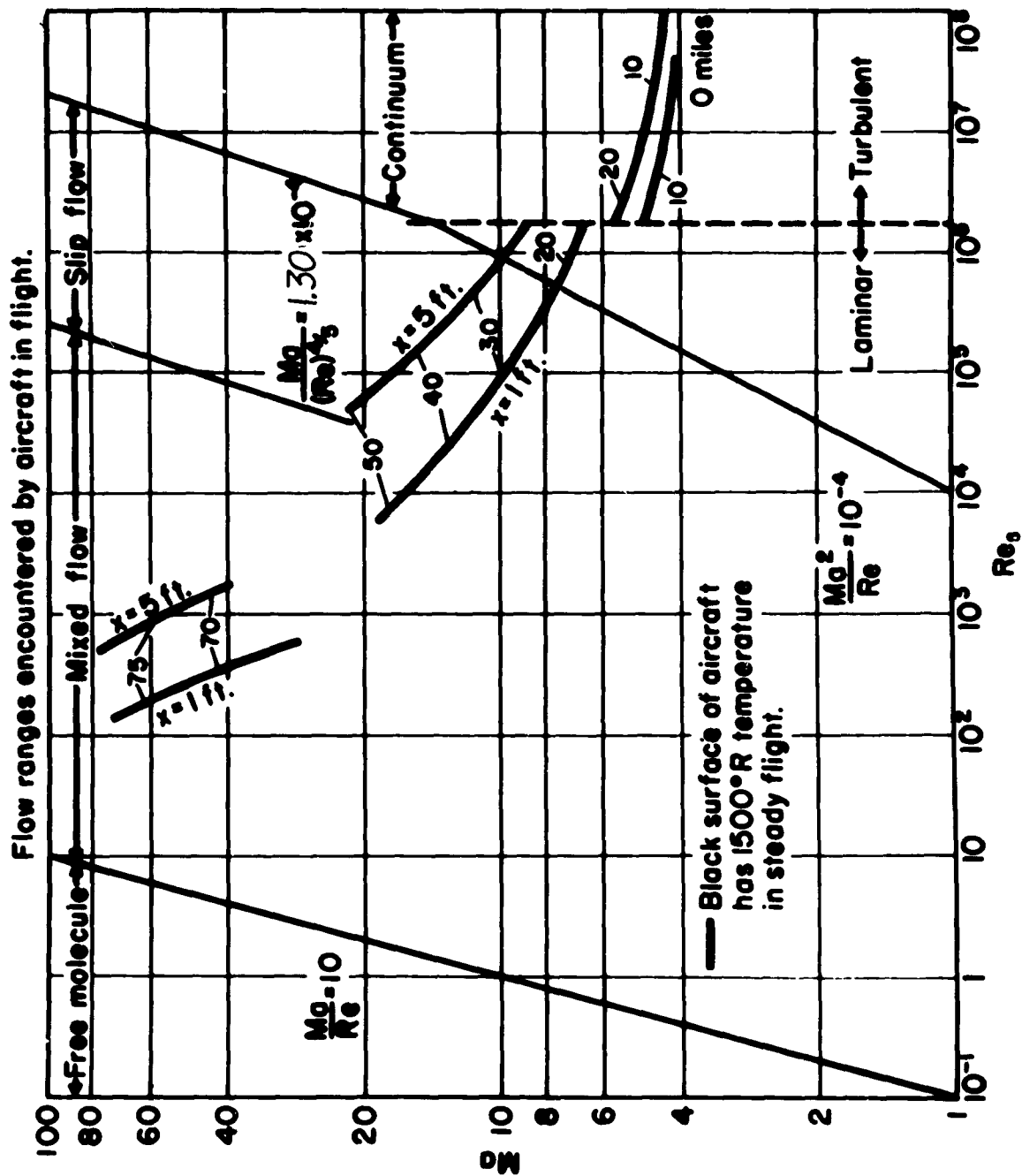


Figure 14. Flow Regimes in Mach-Reynolds Field.



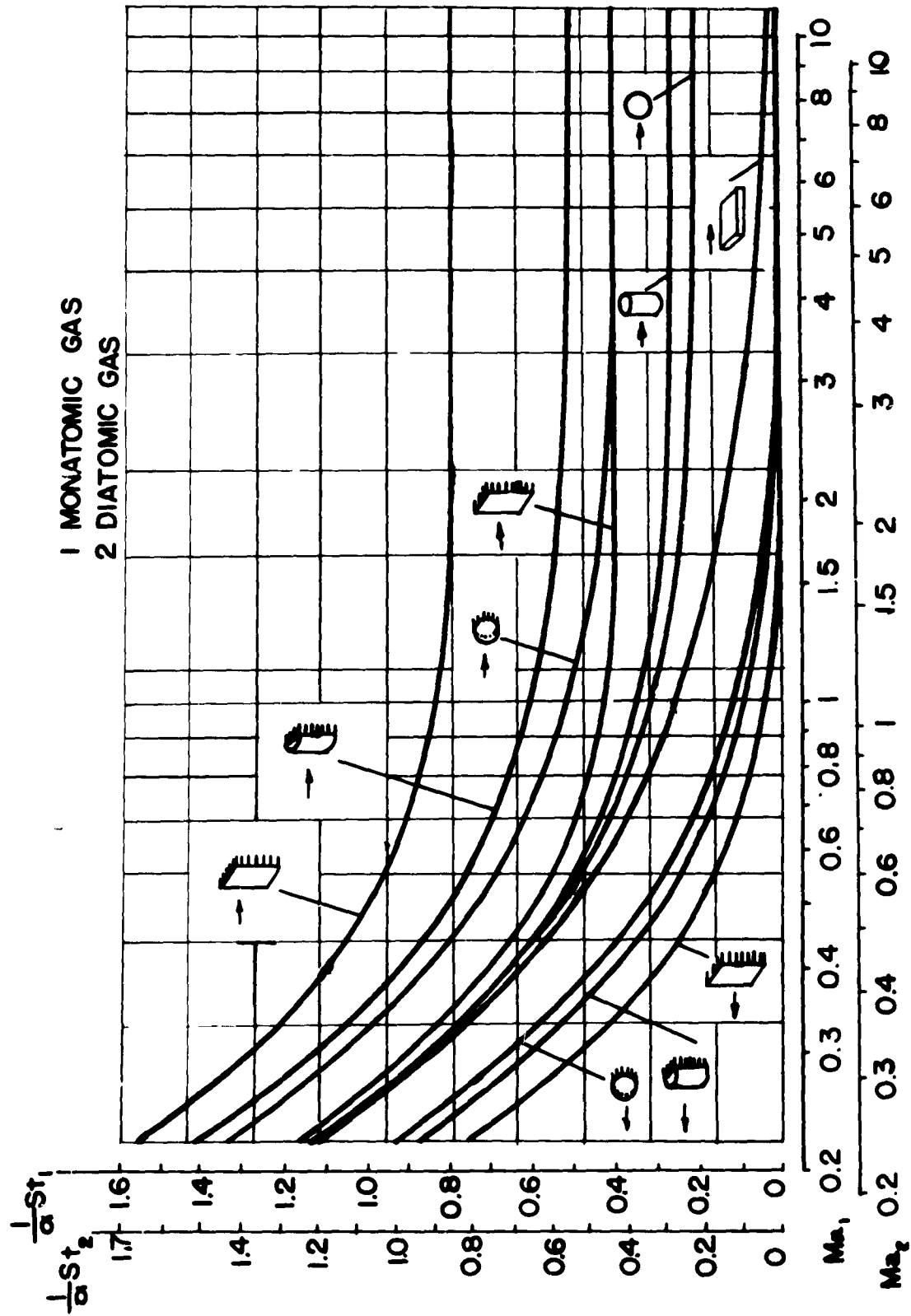


Figure 15. Stanton Numbers for Heat Transfer in Free Molecular Flow Regime

1 MONATOMIC GAS  
2 DIATOMIC GAS

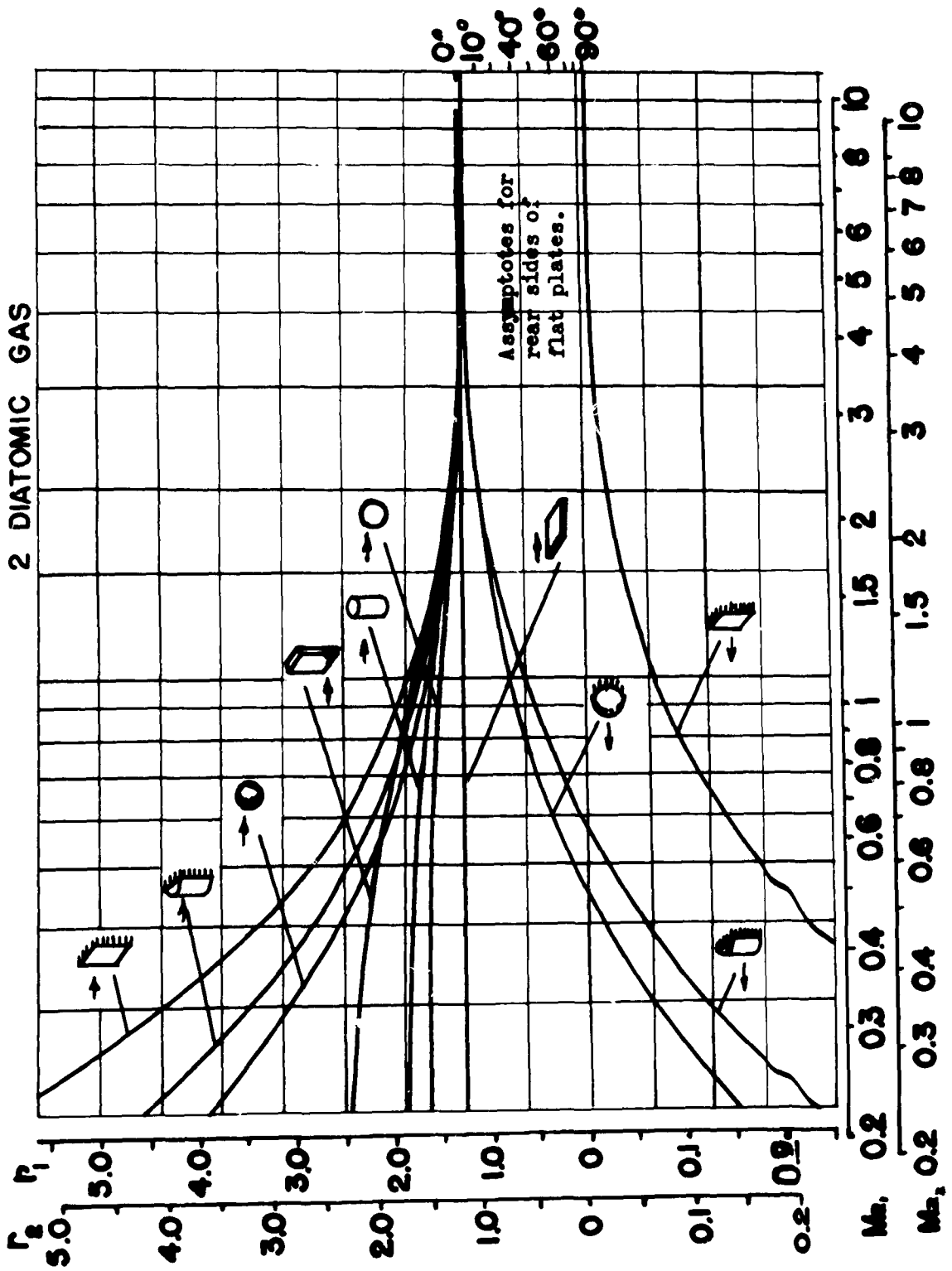


Figure 16. Temperature Recovery Factor for Free Molecular Flow Regime.

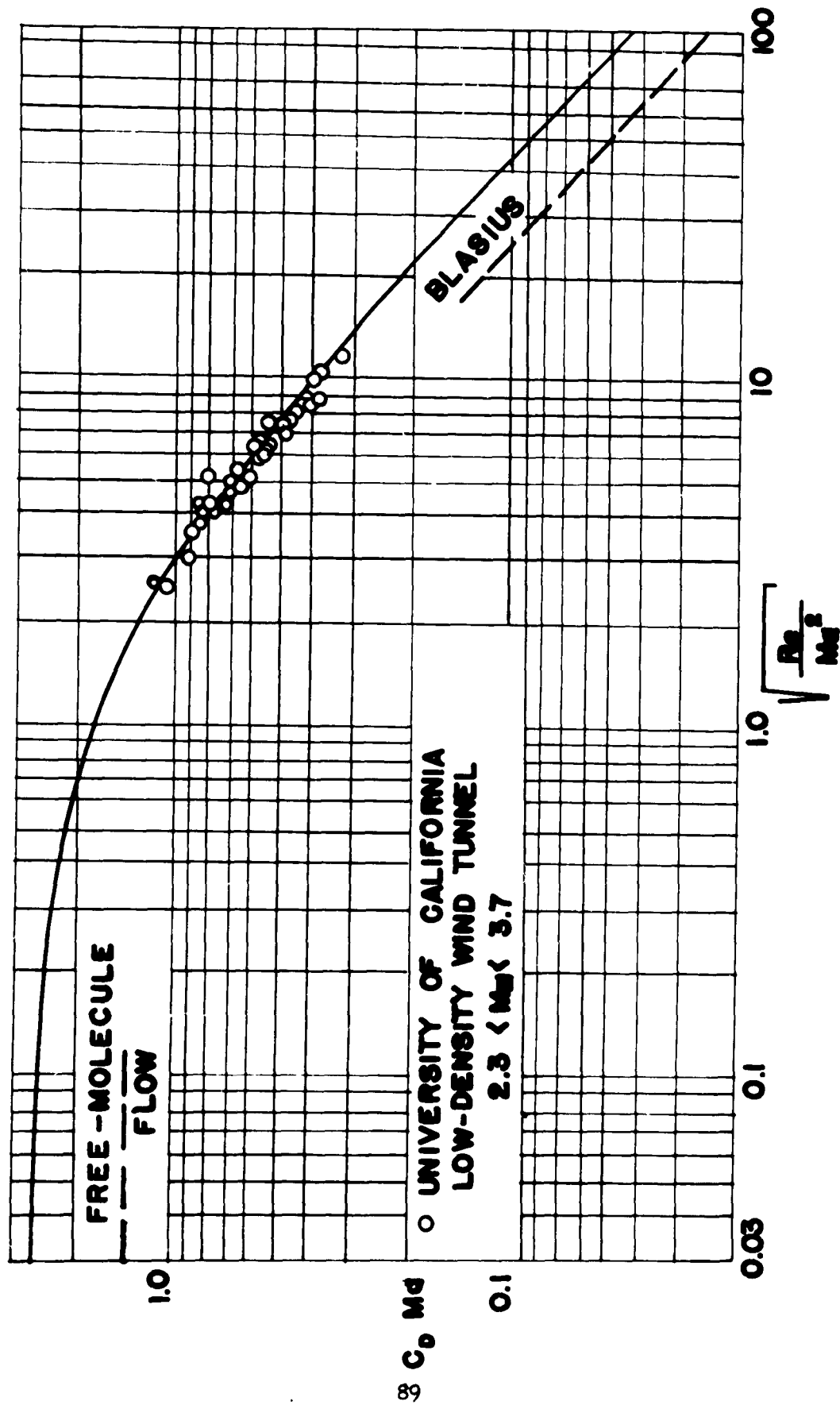


Figure 17. Average Skin Friction Factor for Flat Plate in Rarefied Air.

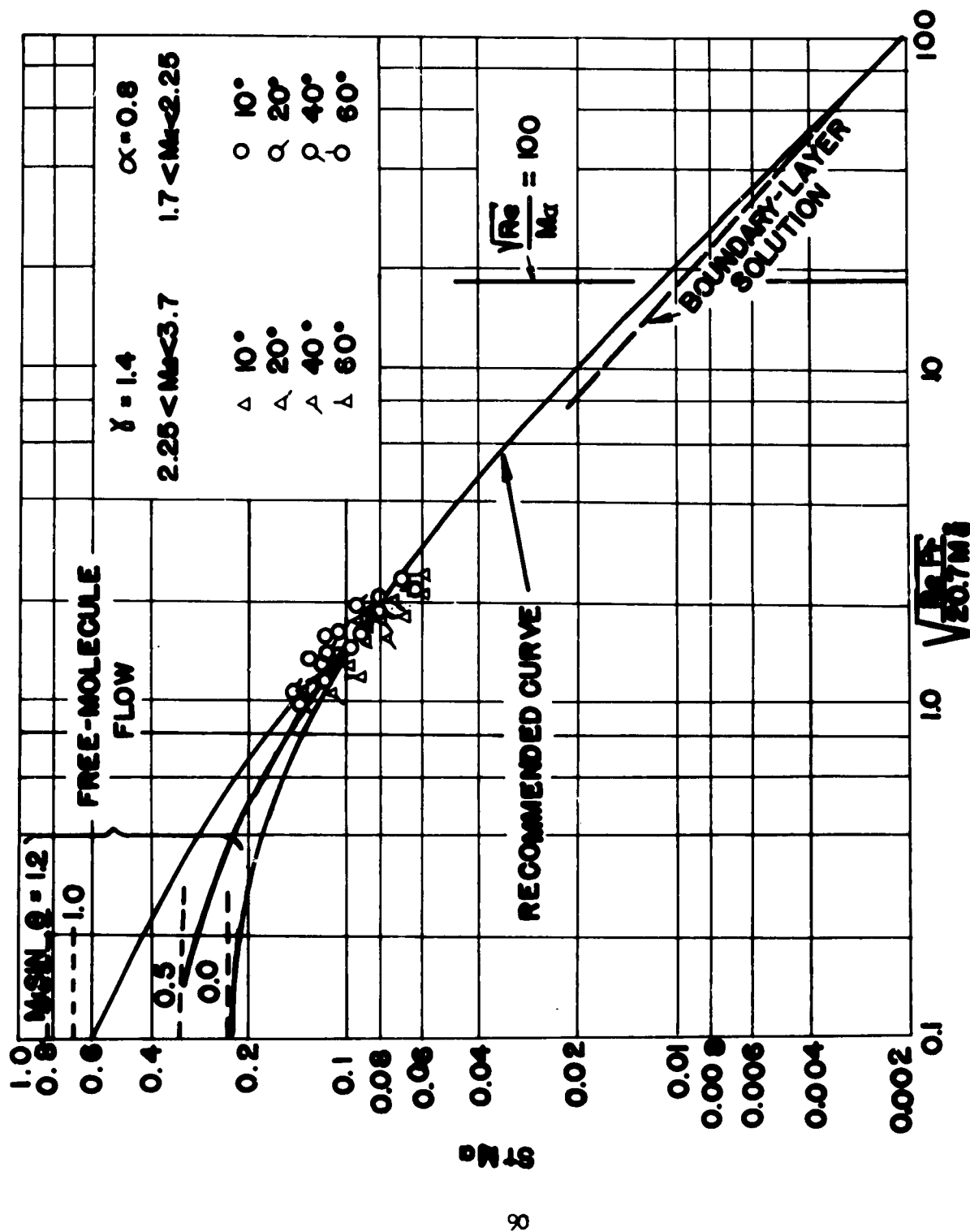


Figure 18. Average Stanton Number for Heat Transfer on Cones in Rarefied Air.

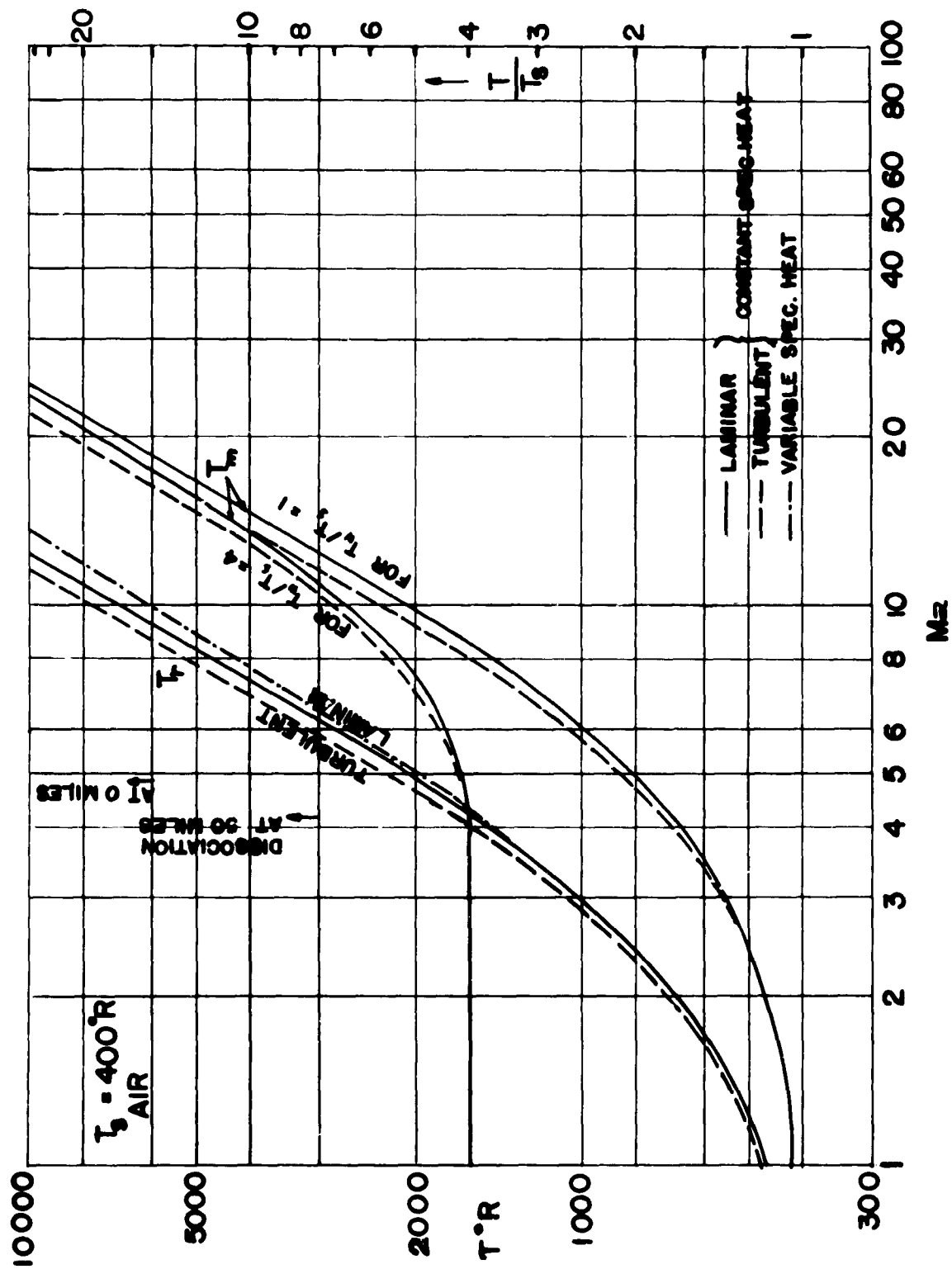


Figure 19. Maximum Temperature  $T_m$  and Recovery Temperature  $T_r$  in Boundary Layer on Flat Plates.

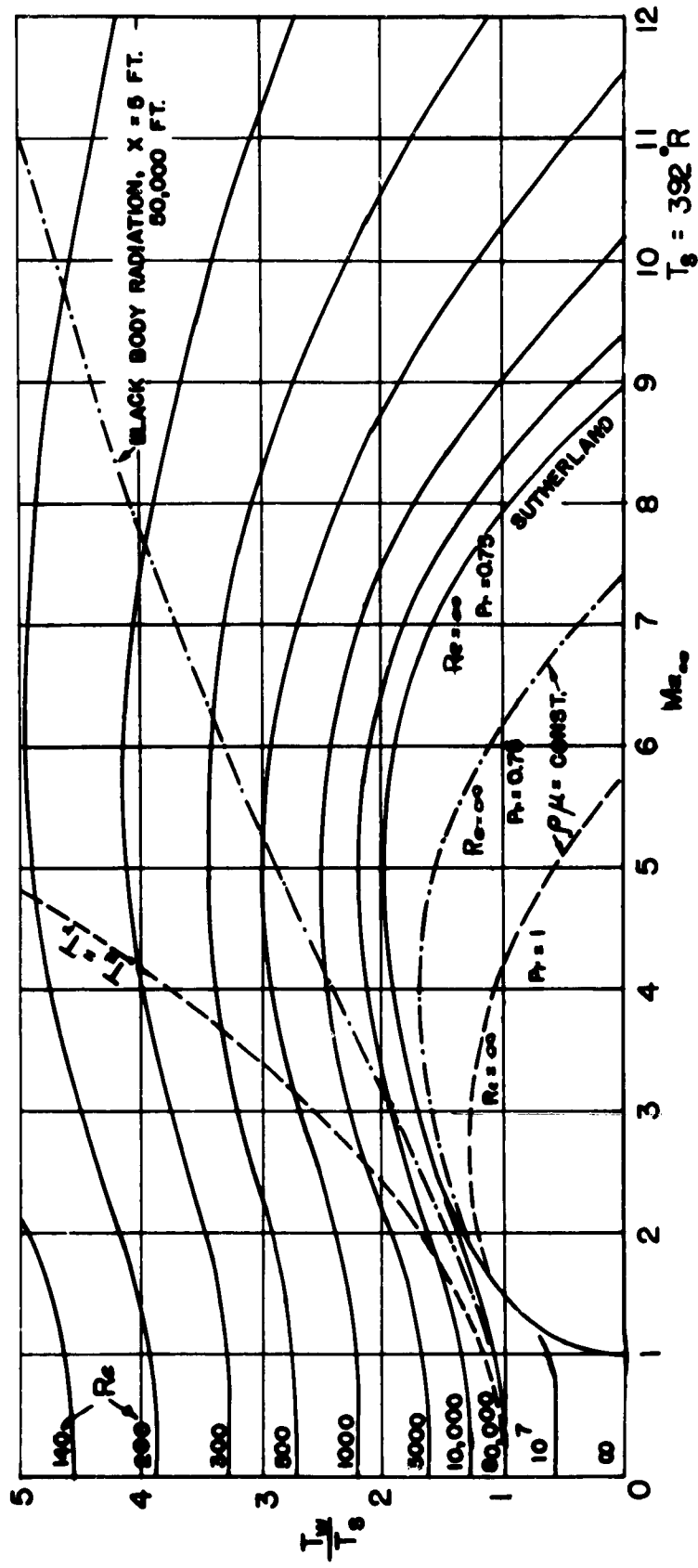


Figure 20. Reynolds Number for Instability of Boundary Layer Flow Along Flat Plate.

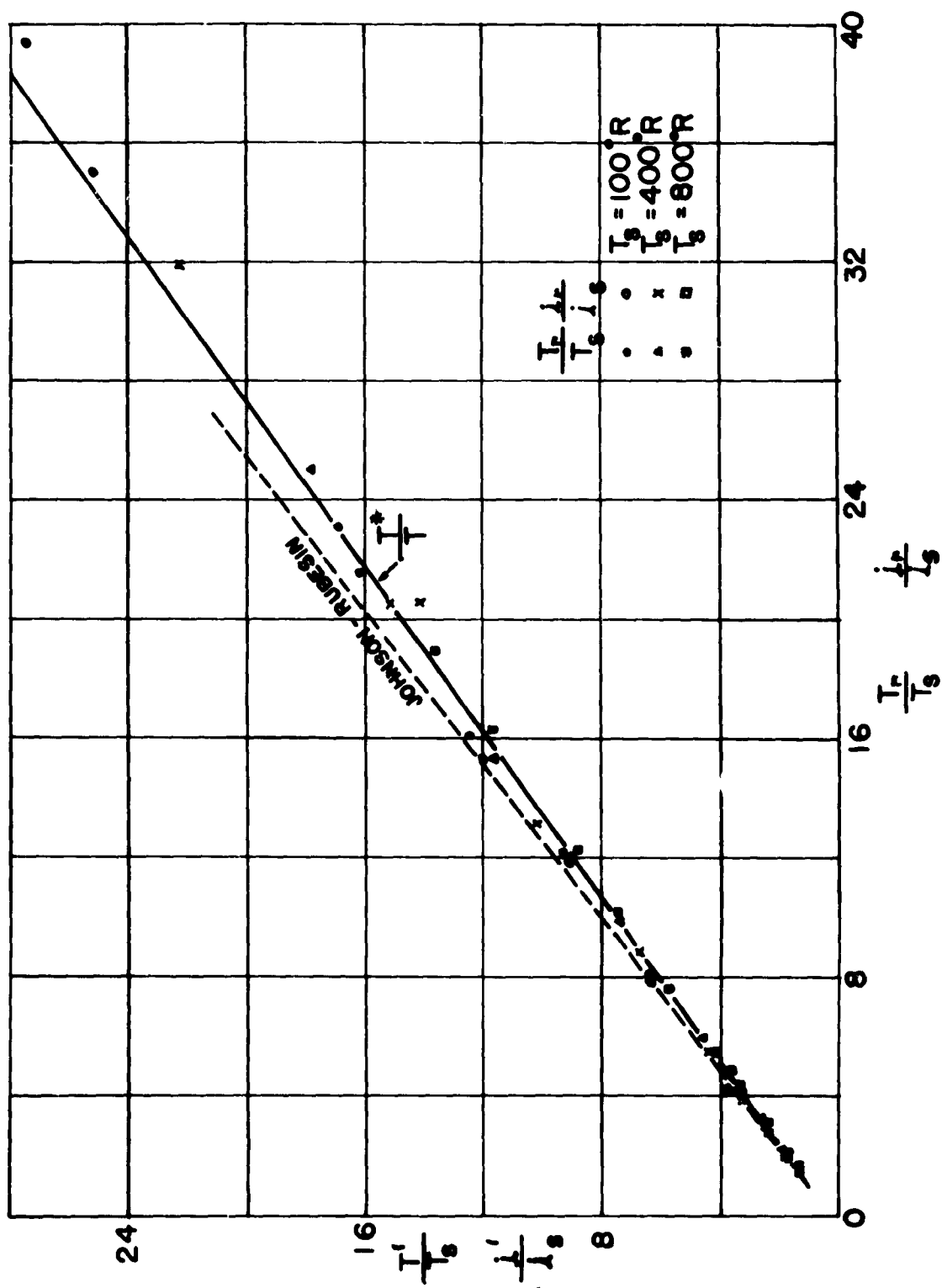


Figure 21. Calculation of Reference Temperature for Laminar Flow Along Flat Plate.

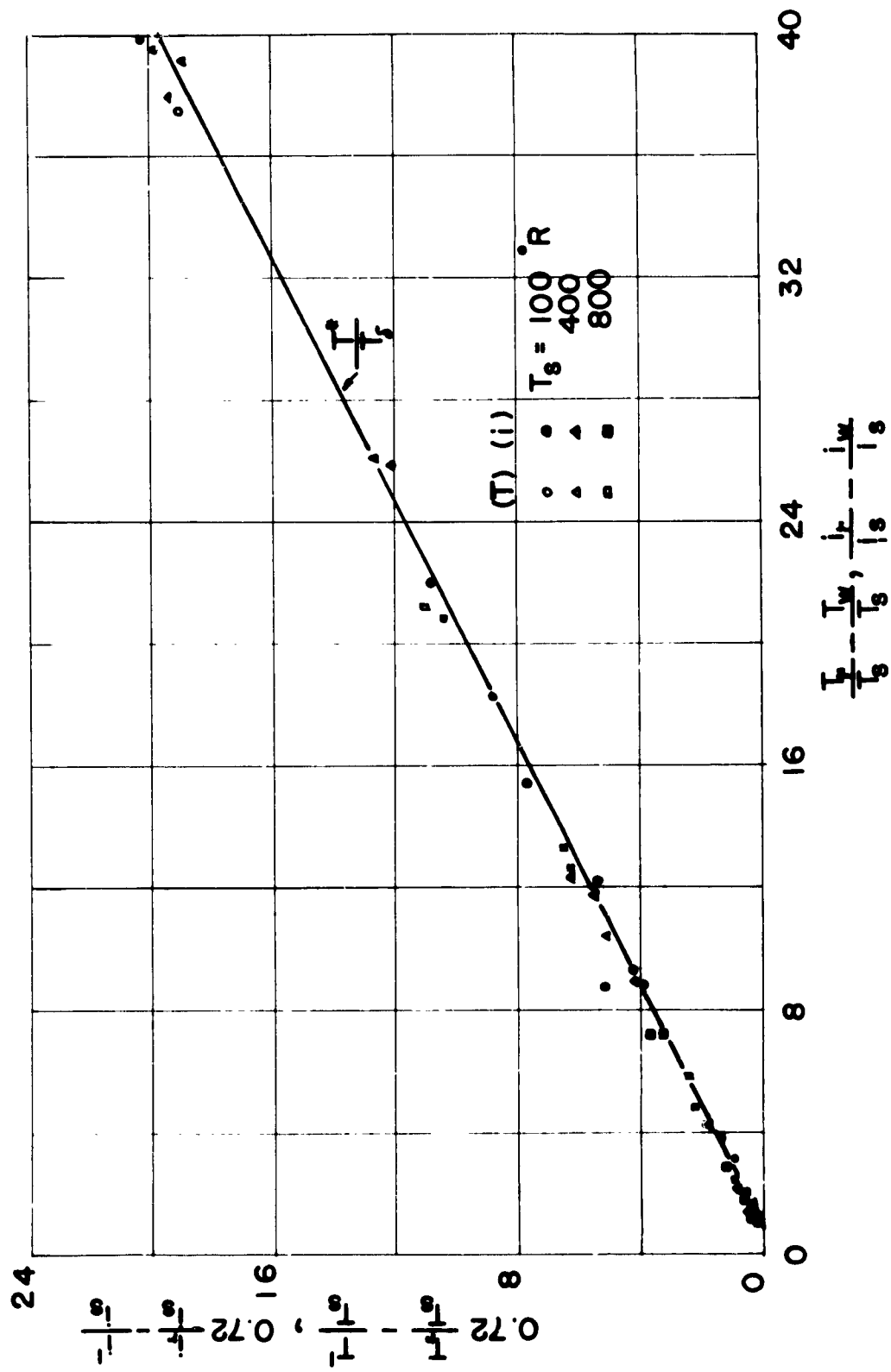


Figure 22. Calculation of Reference Temperature for Laminar Flow Along Flat Plate



<p>University of Minnesota, Minneapolis, Minn. SURVEY ON HEAT TRANSFER AT HIGH SPEEDS by Ernst R. G. Eckert. December 1961. 94 p. incl. illus. tables. (Project 7064; Task 70138) (Contract AF 33(616)-2214) (ARL 189)</p> <p>Unclassified Report This report is a reissue of WADC TR 54-70 which presented a survey of the literature on heat transfer at high supersonic velocities published predominately in the years 1951 to 1954. The large number of papers made it necessary to restrict the survey essentially to surfaces along which the pressure is con- stant, an assumption which is not too restric- tive since evidence could be presented that</p> <p>( over )</p>	<p>UNCLASSIFIED</p>	<p>University of Minnesota, Minneapolis, Minn. SURVEY ON HEAT TRANSFER AT HIGH SPEEDS by Ernst R. G. Eckert. December 1961. 94 p. incl. illus. tables. (Project 7064; Task 70138) (Contract AF 33(616)-2214) ARL 189)</p> <p>Unclassified Report This report is a reissue of WADC TR 54-70 which presented a survey of the literature on heat transfer at high supersonic velocities published predominately in the years 1951 to 1954. The large number of papers made it necessary to restrict the survey essentially to surfaces along which the pressure is con- stant, an assumption which is not too restric- tive since evidence could be presented that</p> <p>( over )</p>	<p>UNCLASSIFIED</p>
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